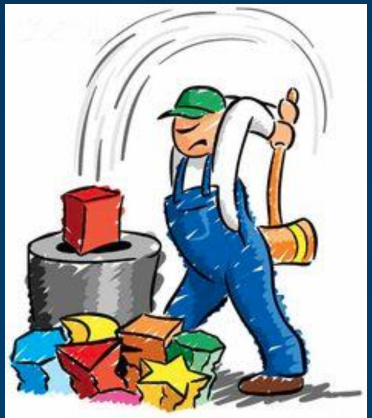
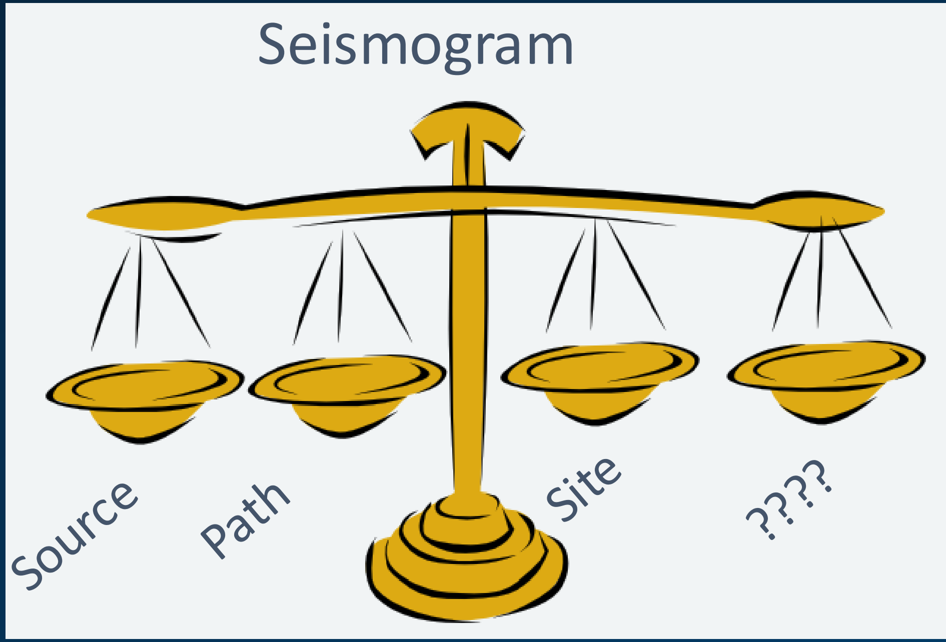
Some thoughts on Earthquake “Stress Drop” and how to calculate it

Rachel Abercrombie (Boston University)
University of Oregon: 17 April 2024



What is it? Why do we care?
How do we try and measure it?
How do we assess estimates?

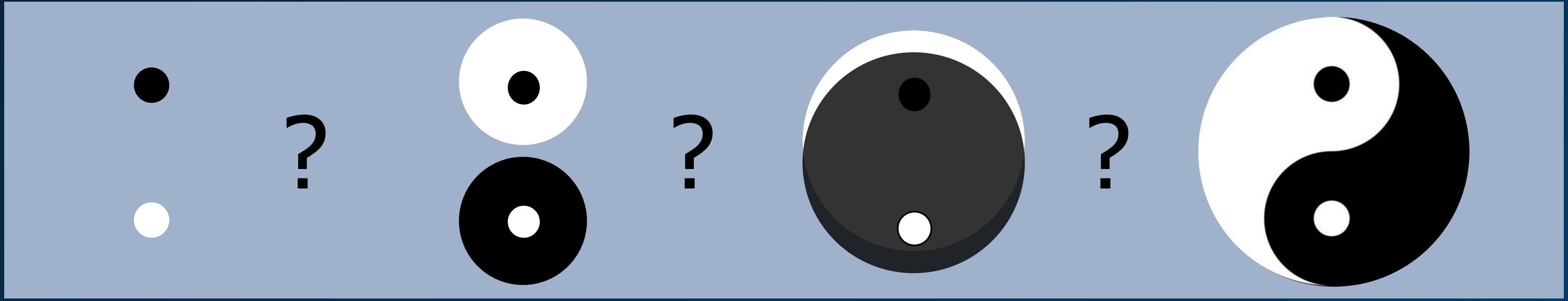


Is it better to find “Best model” or to find range of models that can fit (and ones that cannot)

Resolution, resolution, resolution!

To understand Earthquake interactions, Source physics & Ground motion need to go beyond points

A big difference between → two overlapping/repeating earthquakes and → two adjacent/triggering earthquakes

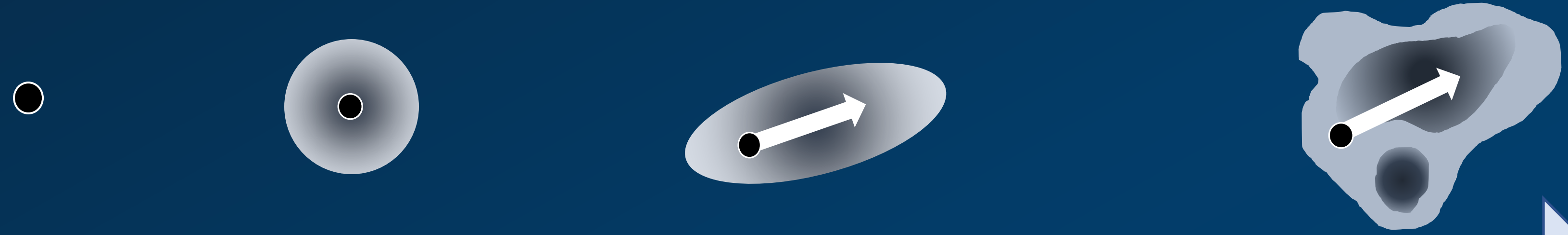


Point

Circle: Radius

Line/Ellipse: length, direction

Complex Slip distribution Asymmetrical, directional rupture

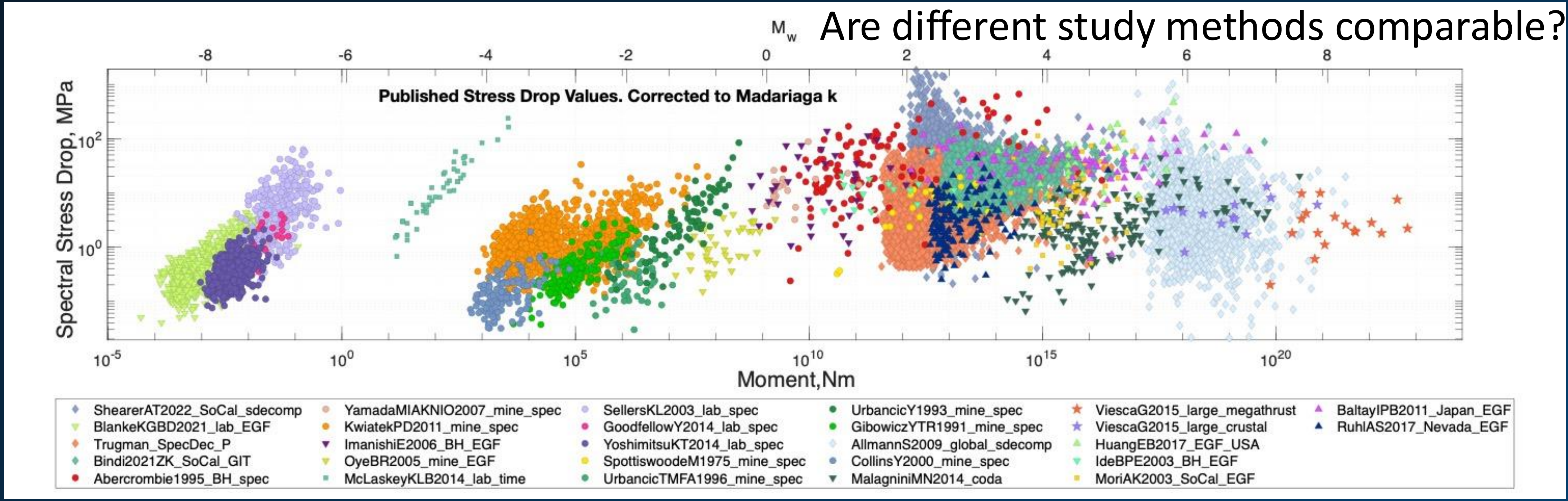


Increasing unknowns and uncertainties needs better data resolution

Rachel Abercrombie: SCEC 2024

Rachel Abercrombie: SCEC 2024

Constant Stress Drop Scaling?



Self-similar process from lab to 100s km,
but some trends in individual studies
Are scatter and trends real?
Or uncertainties?

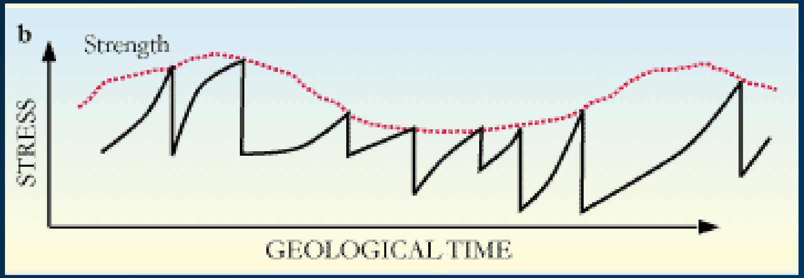
Abercrombie *et al.*, 2017: detailed consistent comparison of distinct tectonic settings in New Zealand – variation within one region >> difference between regions

Seismic Moment:

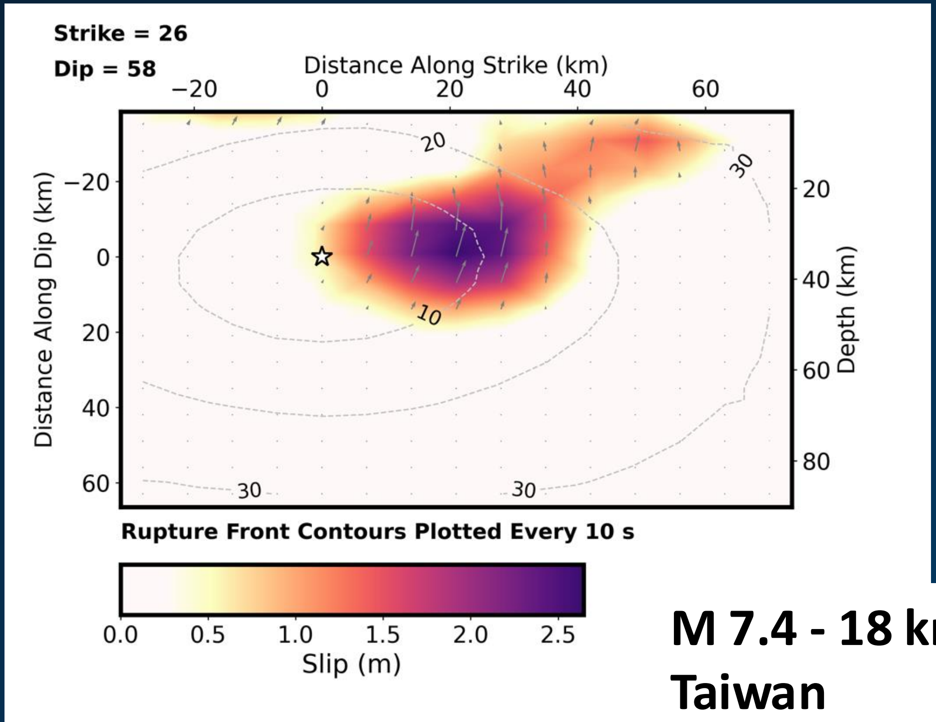
M_0 = rigidity × slip × area

Stress Drop ~ strain ~ slip / $\sqrt{\text{area}}$

What is ”Stress Drop”



Stress drop = stress released over fault as it slips is a commonly used parameter to characterize earthquakes



In an earthquake, a part of the fault slips, releasing stress.

Seismic Moment: M_0 = rigidity × slip × area
= Long period level

“Stress Drop” ~ strain ~ slip / √area

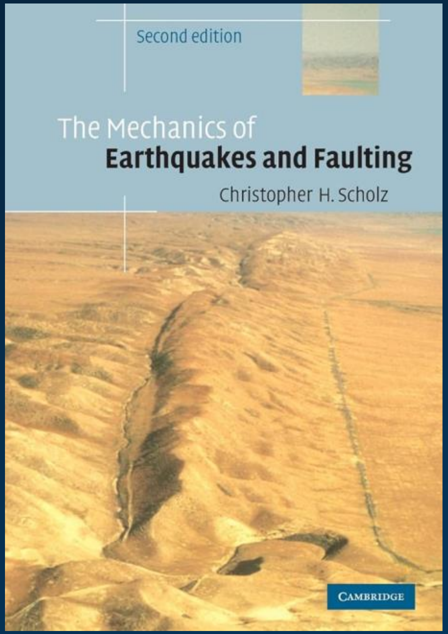
M 7.4 - 18 km SSW of Hualien City, Taiwan

- 2024-04-02 23:58:11 (UTC)
- USGS Model

High slip over small area produces more high frequency shaking than small slip over large area.

Rachel Abercrombie: SCEC 2024

Earthquake Recurrence: slip/time predictable?



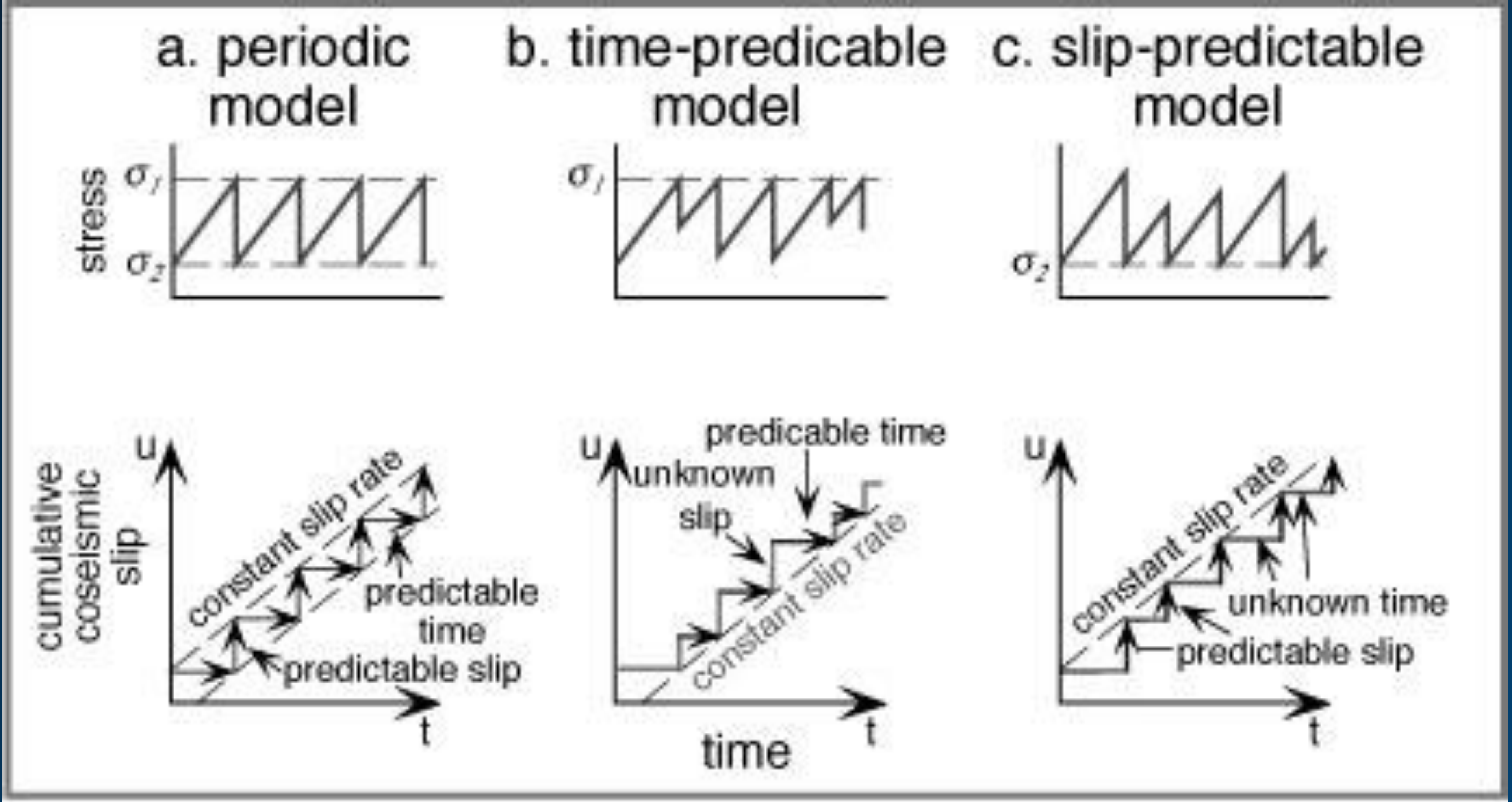


Figure 4.5: Models for earthquake recurrence.

Earthquake Source Models: Crack v. Pulse

- **Crack:** Slip spreads from centre then heals from centre

Simplest class of models. Brune 1970, Madariaga, 1976. Used for least well recorded earthquakes (small)

- **Slip pulse** propagates

Basis of slip inversion models.
Rise time = duration of slip at each point.

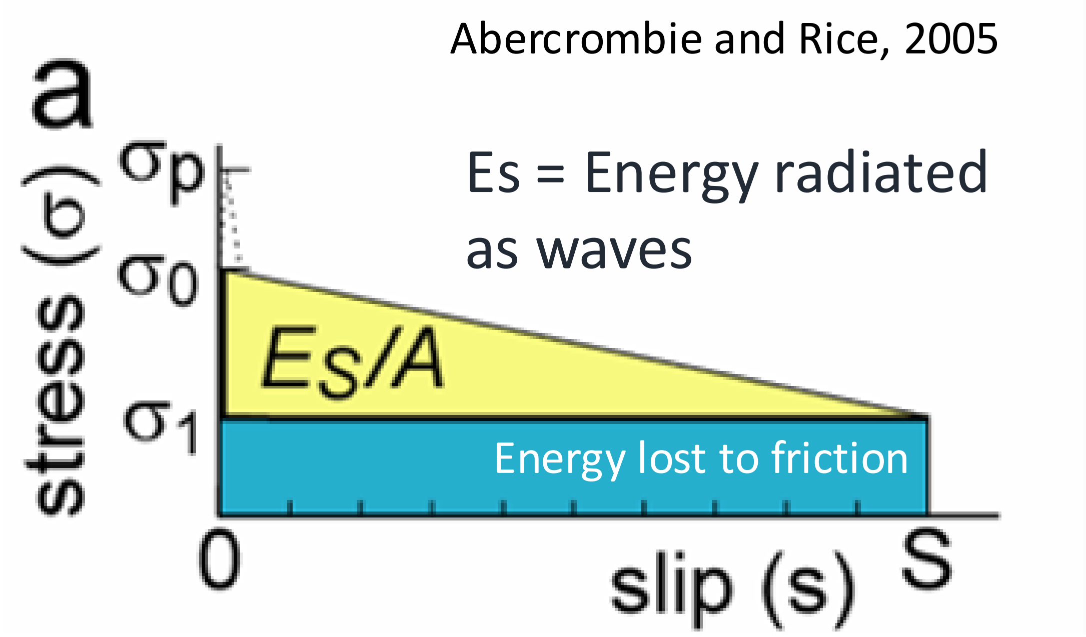
Earthquake Energy Budget: Slip at a point

Stress & slip at each point

σ_0 initial stress σ_1 final static stress

σ_P peak stress $\sigma_F(S)$ final dynamic stress

Friction drops instantaneously at start of sliding



Stress drop

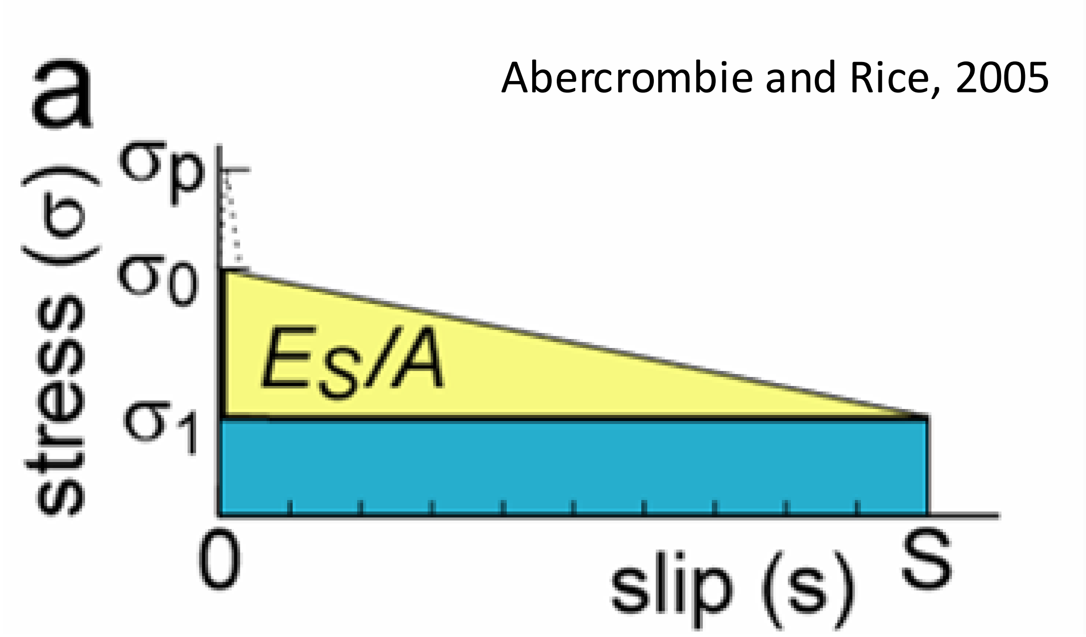
$= \sigma_0 - \sigma_1$

= stress at start – stress at end

Earthquake Energy Budget: Slip at a point

Simplest case:

$$E_S = \frac{1}{2}\Delta\sigma\overline{\Delta u}A$$



More generally:

$$\sigma_a = \mu\frac{E_S}{M_0}$$

Apparent Stress

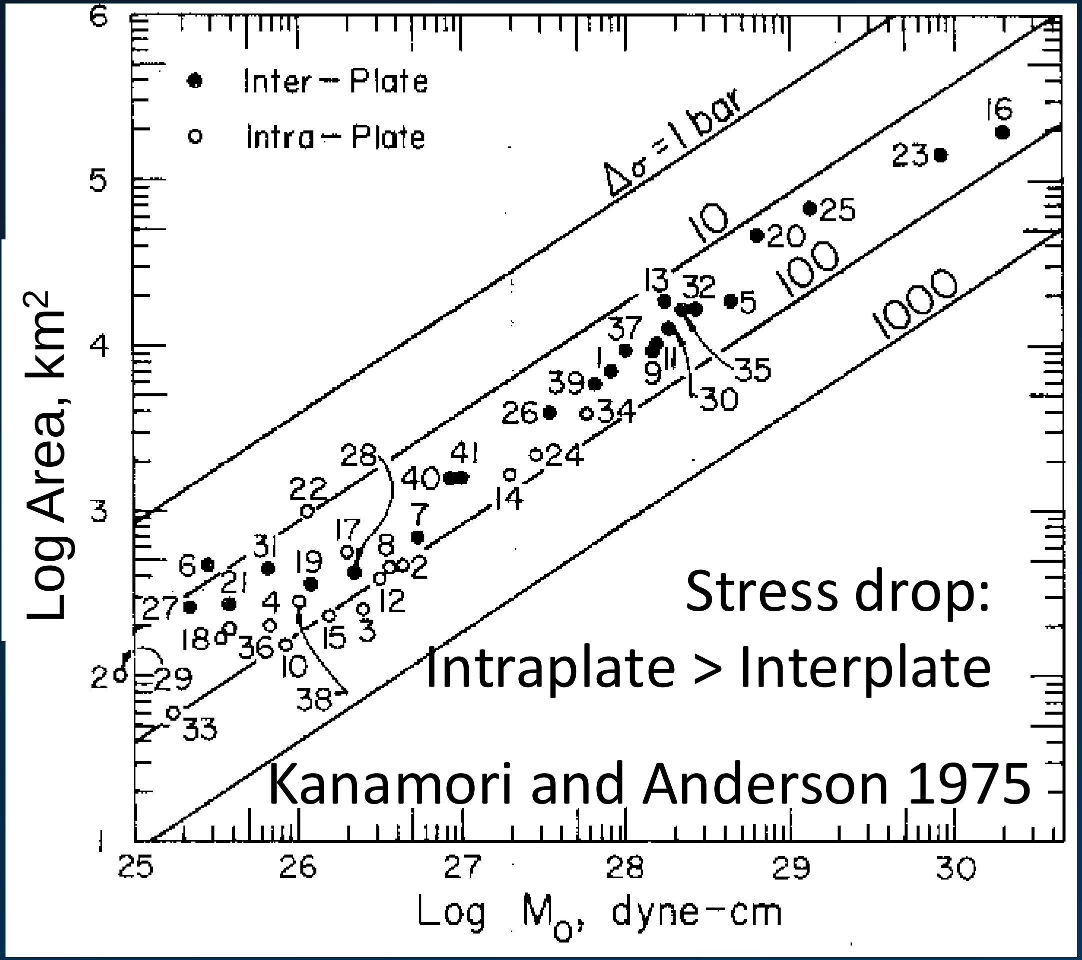
Efficiency:

$$\eta = \frac{E_S}{W_f}$$

$$\eta = \frac{\Delta\sigma}{\sigma_1 + \sigma_2}$$

Friction drops instantaneously at start of sliding

Do Earthquake Sources depend on tectonic setting?



Hypotheses:

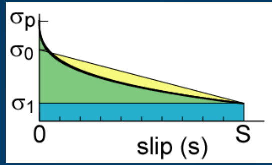
Stress release controlled by tectonic setting

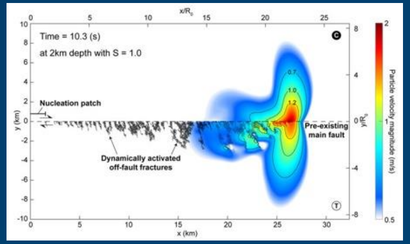
Higher stress drop earthquakes in high stress settings (deeper, intraplate - longer healing, reverse faulting)

Lower stress drop earthquakes in lower stress settings (shallower, interplate, normal faulting

Scaling? Stress drop large = stress drop small?

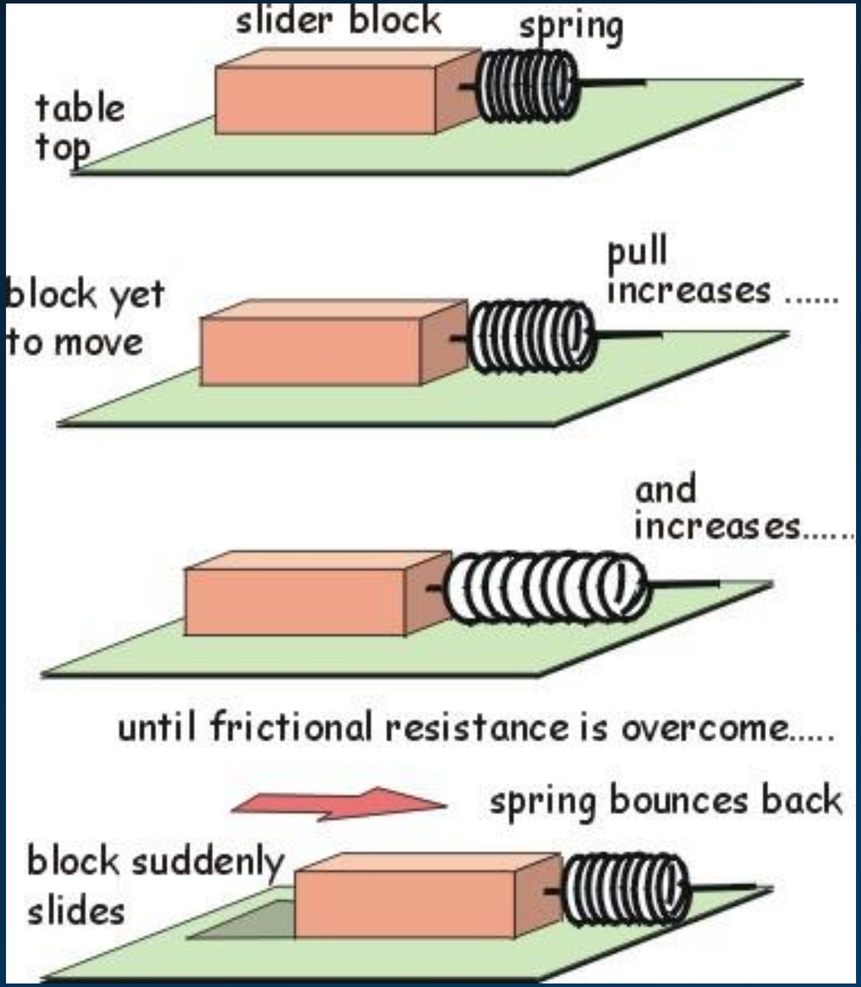
What about induced earthquakes? Same?





Rachel Abercrombie: SCEC 2024

Sliding Block Models, Lab, and Rate & State Friction



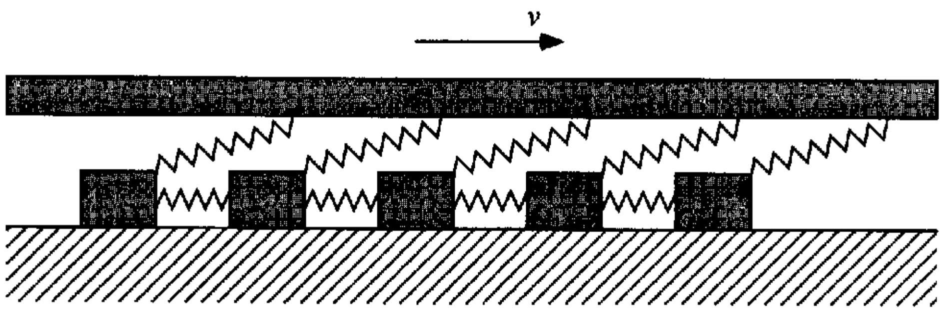
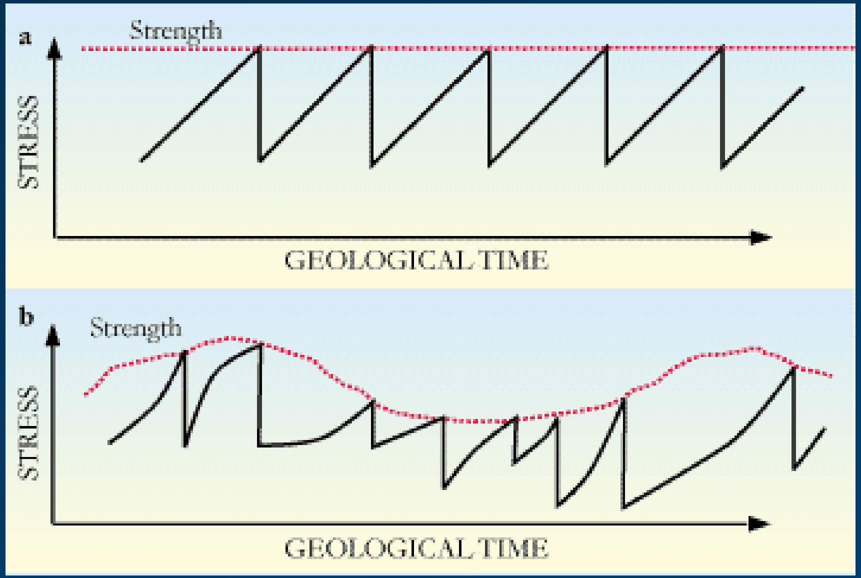


Fig. 10.4. The sliding block model for simulating earthquakes.

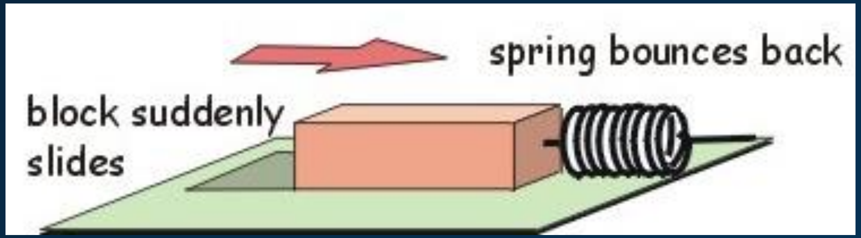


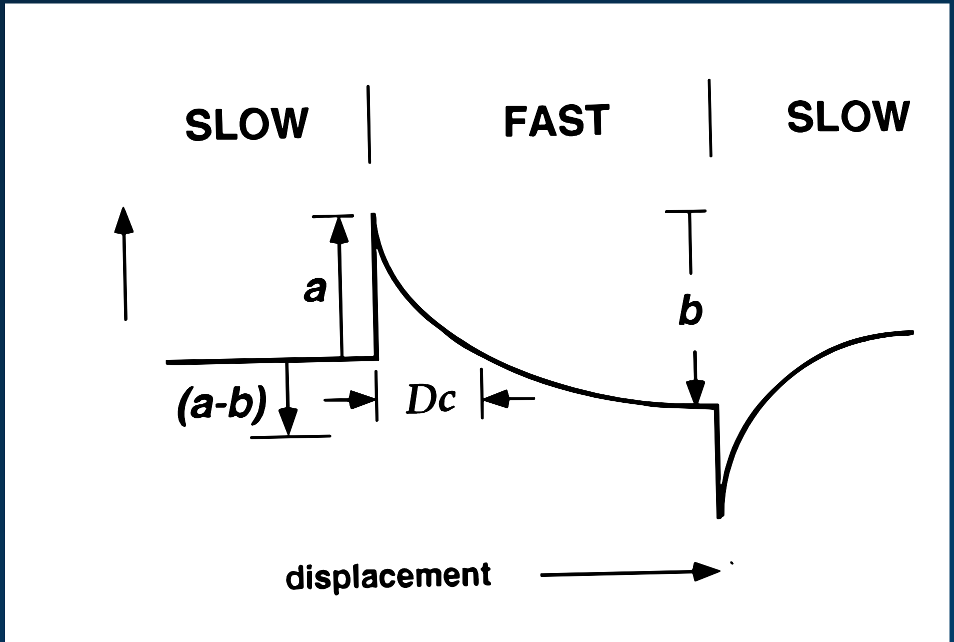
Rachel Abercrombie: SCEC 2024

Sliding Block Models, Lab, and Rate & State Friction

What happens during sliding?

Dieterich-Ruina Constitutive Relation: Rate and State Friction

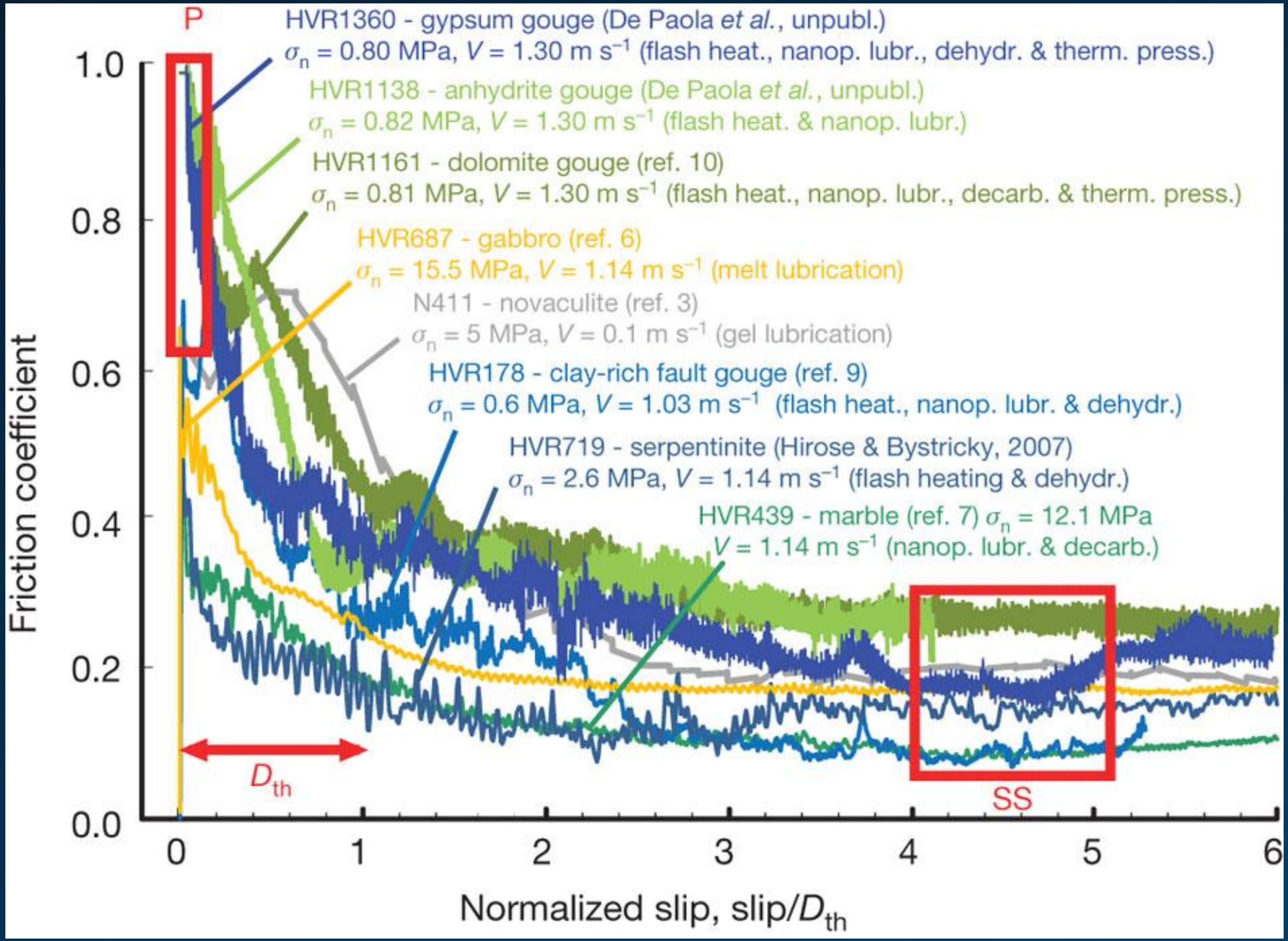




$$\mu = \mu(V,\theta) = \mu_0 + a\ln\left(\frac{V}{V_0}\right) + b\ln\left(\frac{V_0\theta}{D_c}\right)$$

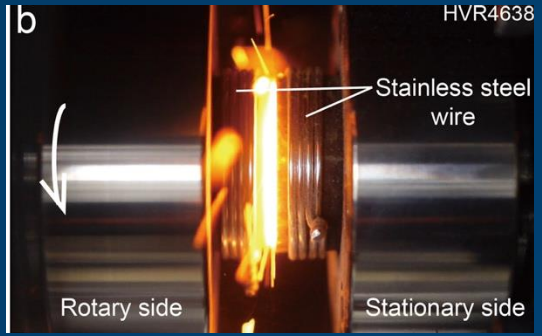
$$\dot{\theta} = 1 - \left(\frac{V\theta}{D_c}\right)$$

Fault lubrication during Earthquakes Di Toro *et al.*, Nature 2011



How does μ decrease?
Melting?
Flash heating?
elastohydrodynamic lubrication?
silica gel formation?

How big is D_c?
Lab: ~10 μm, EQ: ~cm-m
Who cares?



Rachel Abercrombie: SCEC 2024

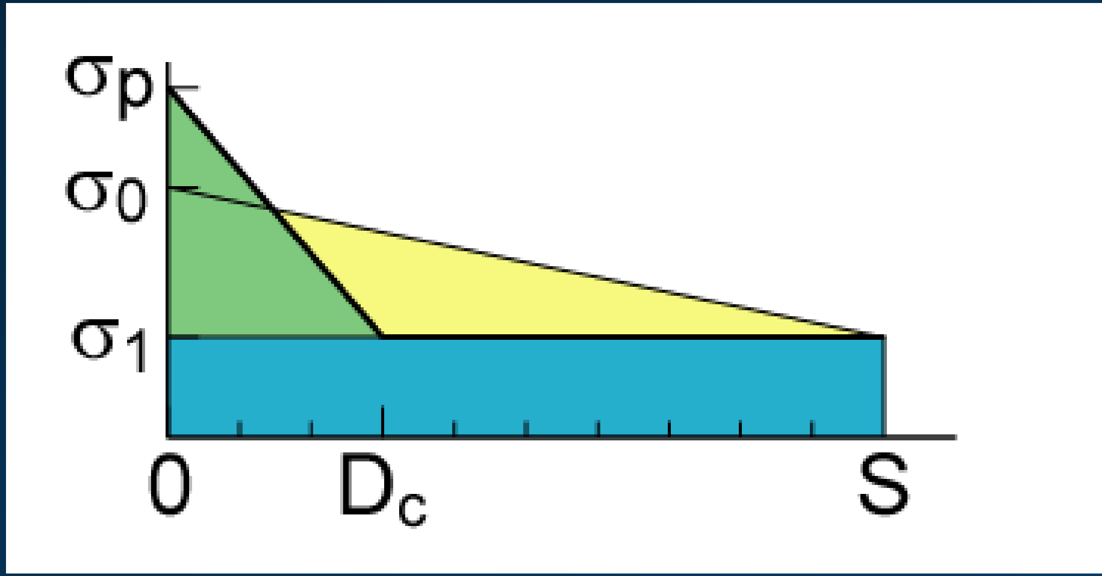
Earthquake Energy Budget: Slip at a point

Simplest case:

$$E_S = \frac{1}{2}\Delta\sigma\overline{\Delta u}A$$

Friction drops over characteristic slip distance, Dc

Abercrombie and Rice, 2005



Now added term called Fracture Energy or Breakdown Work

Fracture Energy (*G*)

Frictional Energy (*F/A*)

Seismic energy:

$$\frac{E_S}{A} = (\sigma_0-\sigma_1)\frac{S}{2} - G$$

Rachel Abercrombie: SCEC 2024

Earthquake Energy Balance: Scaling

When does an earthquake "know" how big it will be? (prediction)

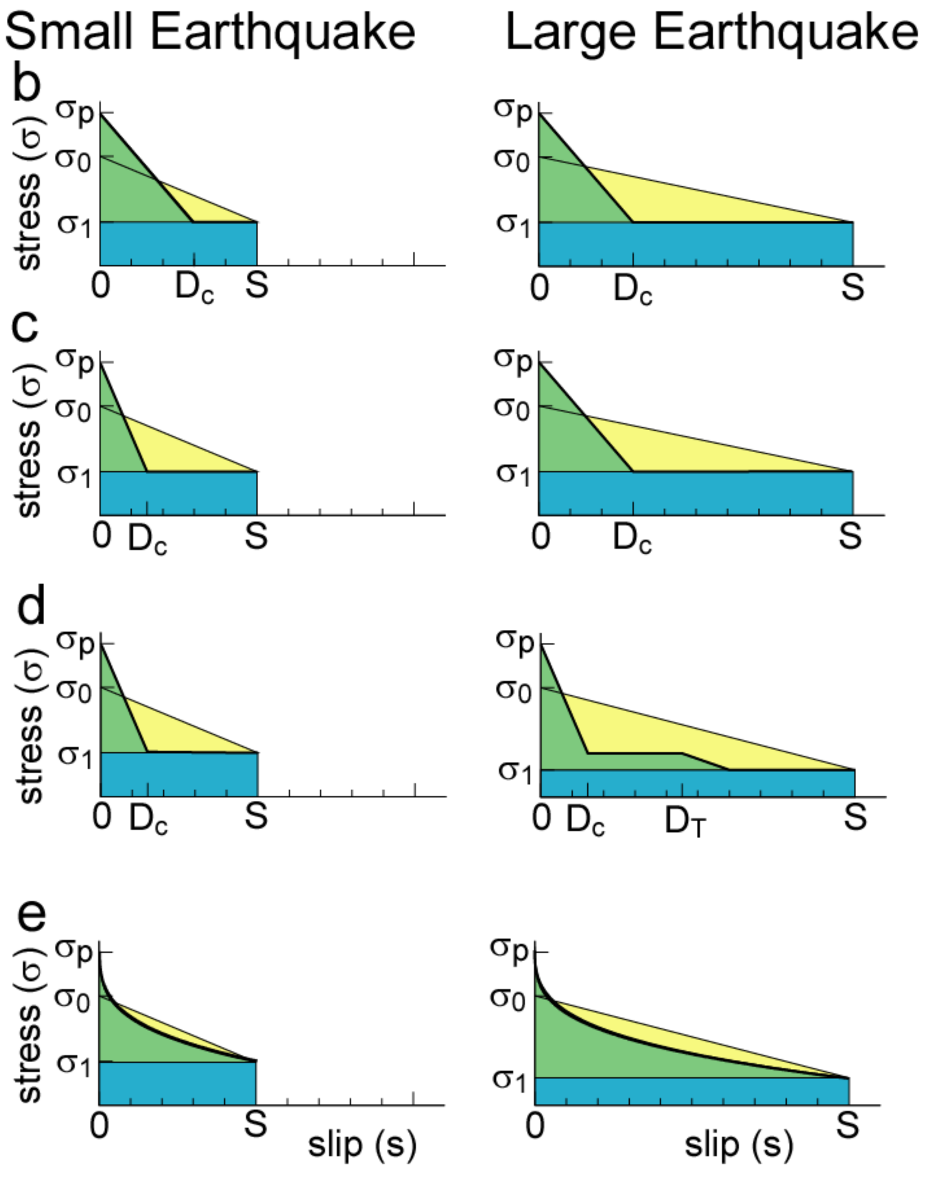
Abercrombie and Rice, 2005

Slip weakening distance D_c independent of earthquake size

Slip weakening distance D_c increases with earthquake final slip

Constant initial weakening, then second weakening (melting?) at large slips Kanamori & Heaton (2002)

Non-linear slip weakening relation - rate of weakening unrelated to final slip



Fracture Energy (G)

Frictional Energy (F/A)

Seismic energy:

$$\frac{E_S}{A} = (\sigma_0 - \sigma_1)\frac{S}{2} - G$$

Rachel Abercrombie: SCEC 2024

But what if Final Static stress ≠ final dynamic stress?

Overshoot or Undershoot

(= Partial Stress drop)

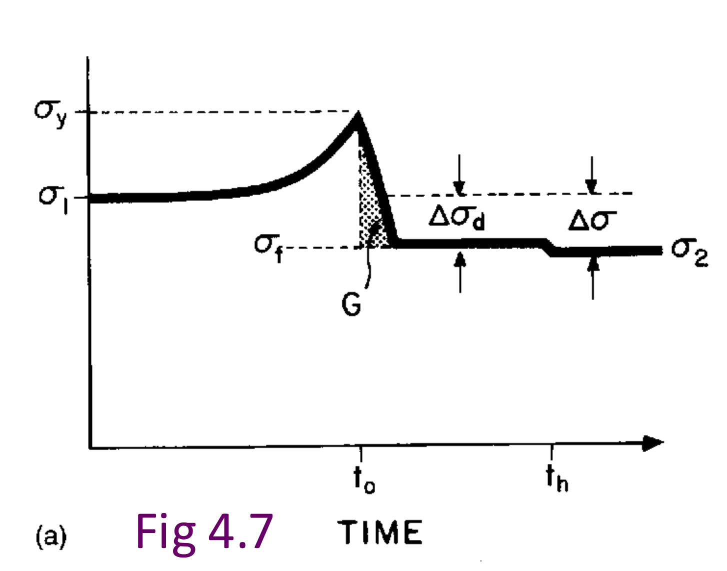
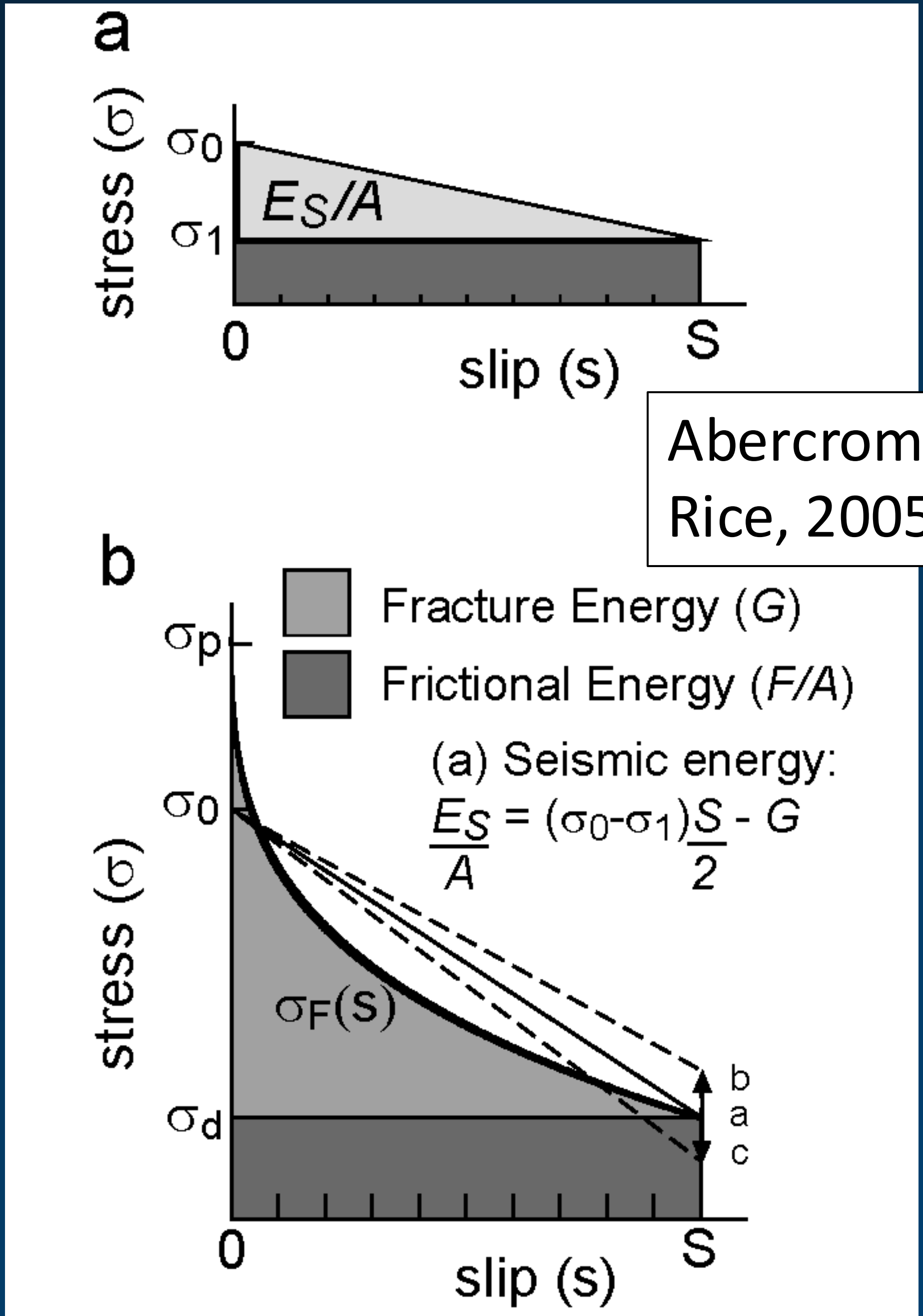


Fig 4.7



Abercrombie and Rice, 2005

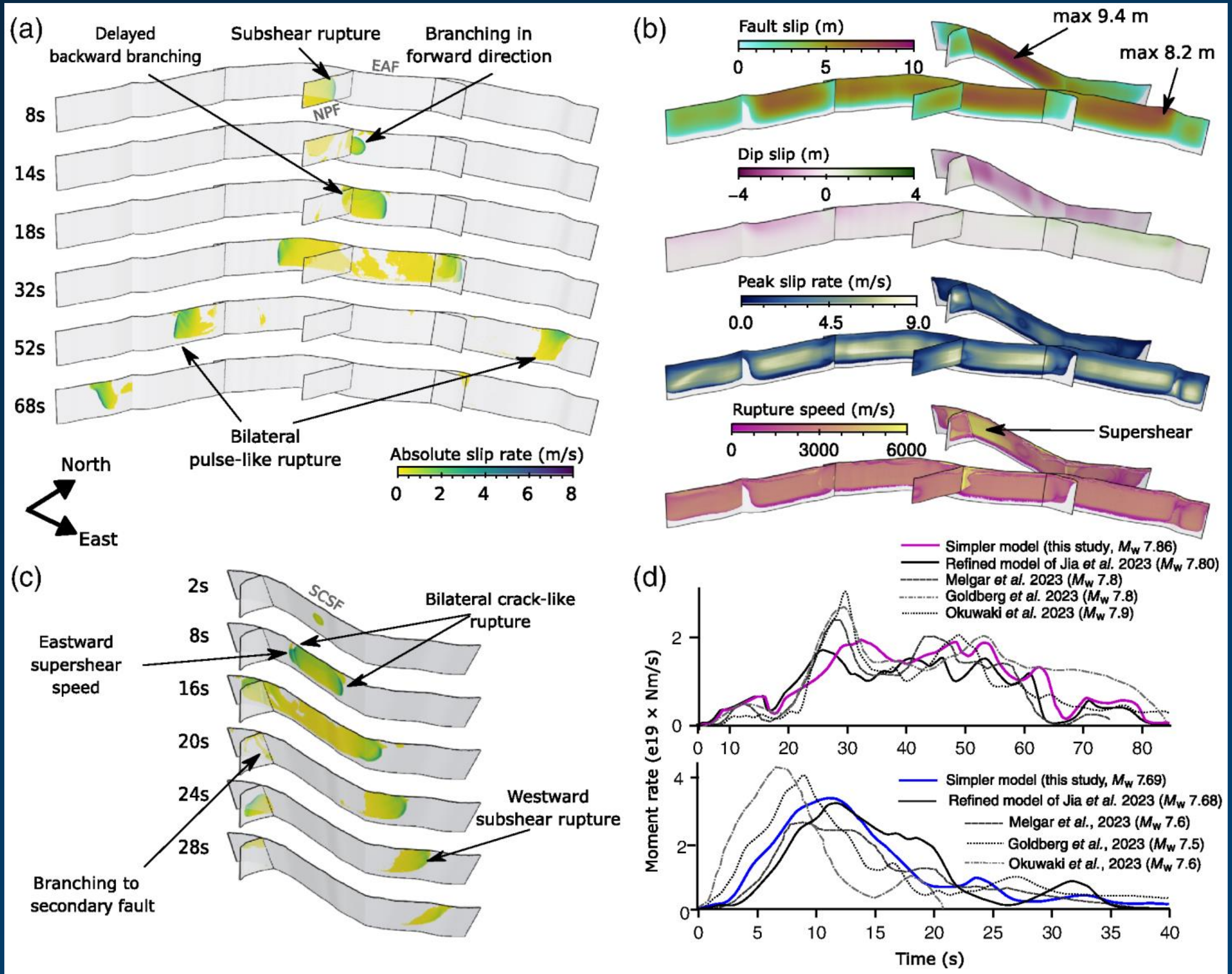
Rachel Abercrombie: SCEC 2024

Dynamic Modelling - needs many assumptions

Geometry, prestress, dynamic friction parameters etc: depend on quality of observational constraints

Note: How big is a "point"?

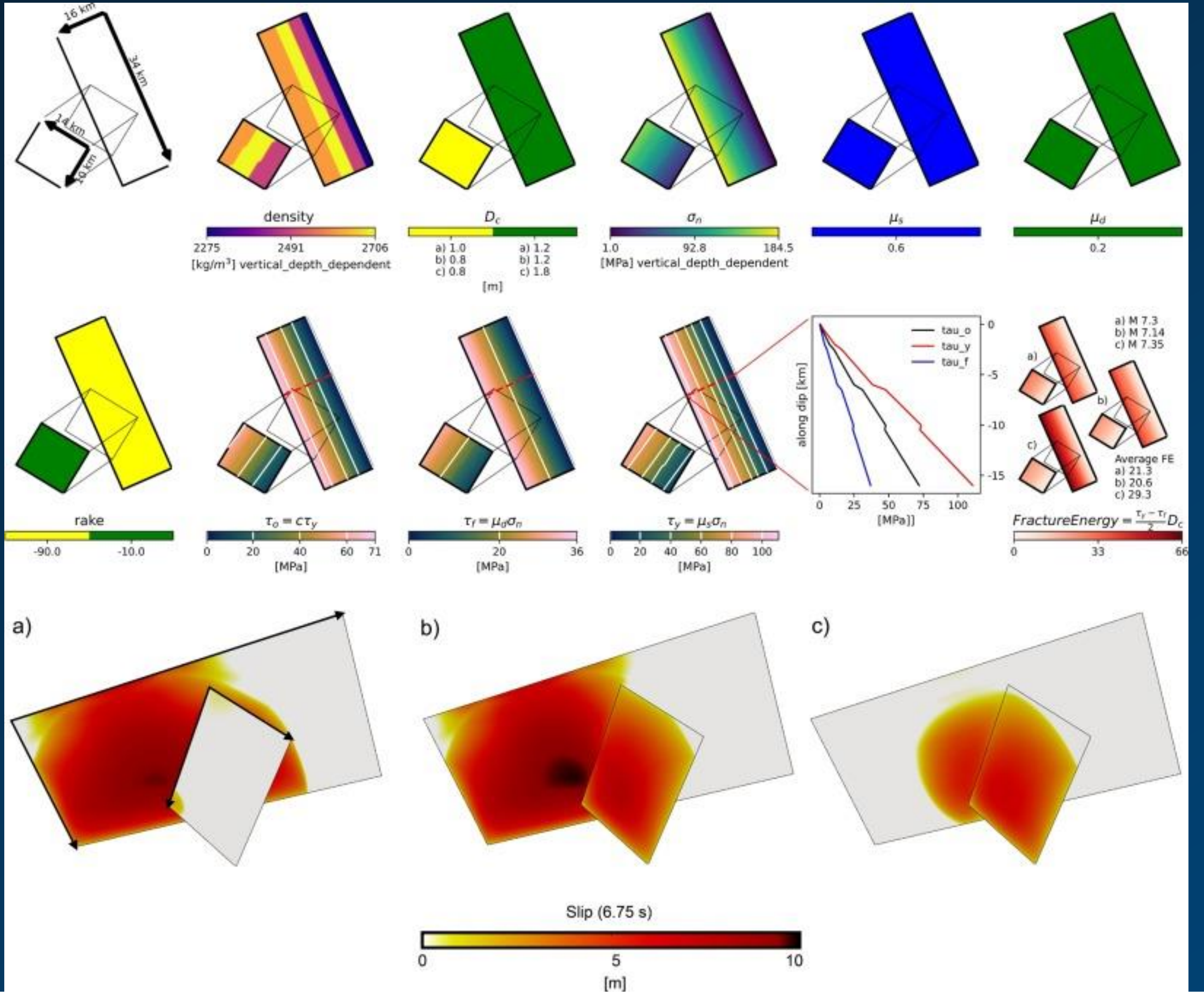
E.g. Gabriel *et al.*, 2023: Feb 2023 Turkey Earthquakes (*Dc* 0.5-1 m)



Dynamic Modelling - needs many assumptions

Geometry, prestress, dynamic friction parameters etc: depend on quality of observational constraints

E.g. Tinti et al. EPSL 2021

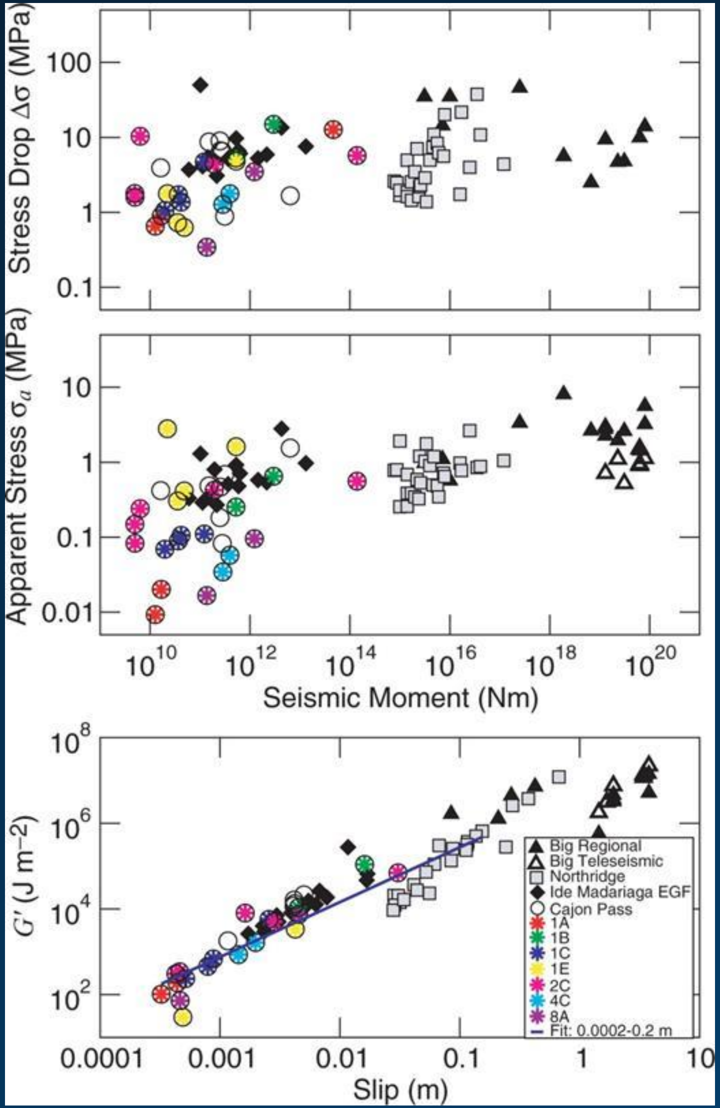


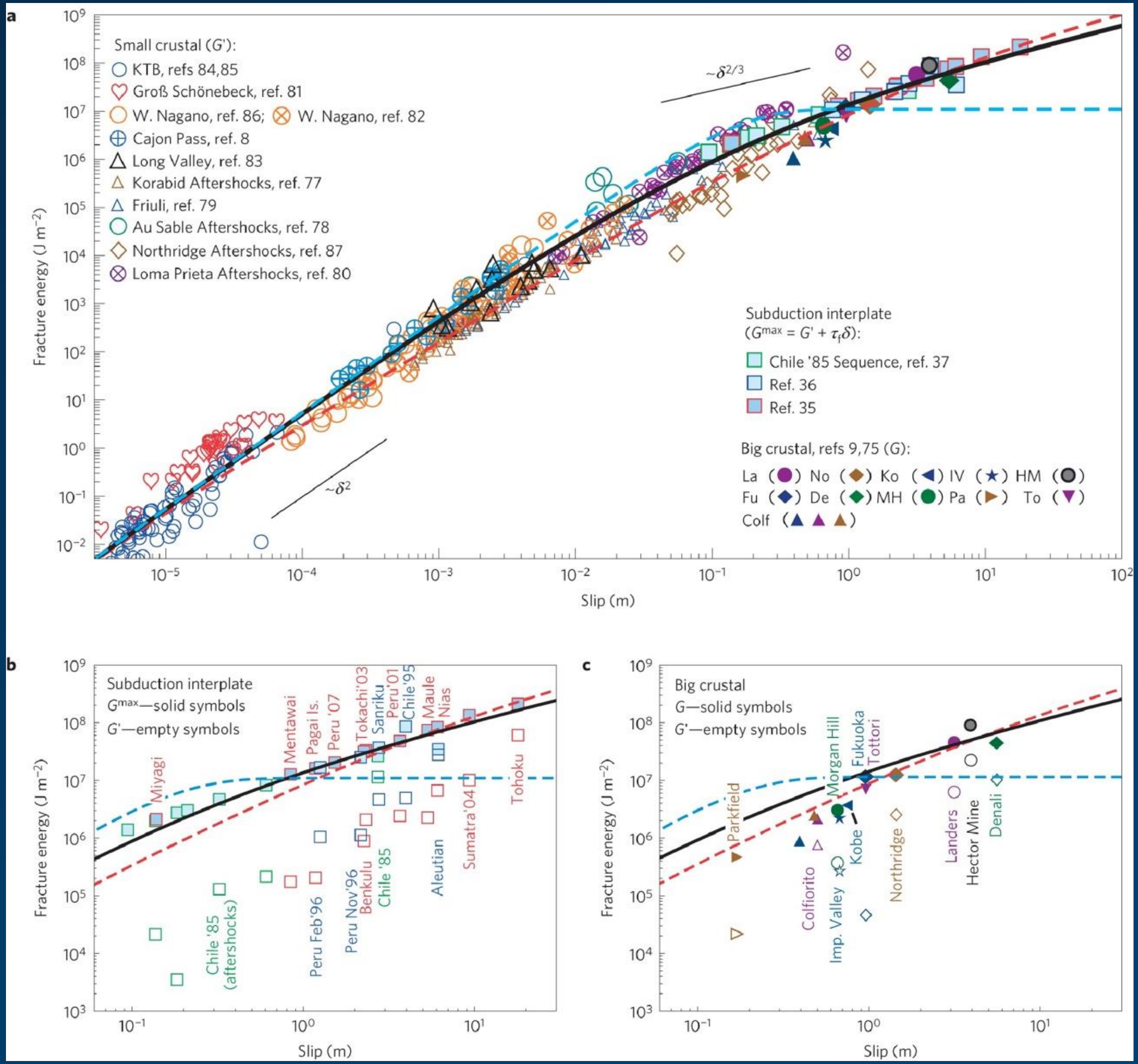
planes (Family (Hom)). Bottom panels: slip distribution after 6.75 s of rupture initiation for models with: a) $D_c^{F155} = 1.2$ m and $D_c^{F210} = 1.0$ m; b) $D_c^{F155} = 1.2$ m and $D_c^{F210} = 0.8$ m; c) $D_c^{F155} = 1.8$ m and $D_c^{F210} = 0.8$ m. The

Estimating and Modelling Fracture Energy and Dc

Viesca & Garagash, 2015: Ubiquitous weakening of faults due to thermal pressurization

Abercrombie & Rice, 2005

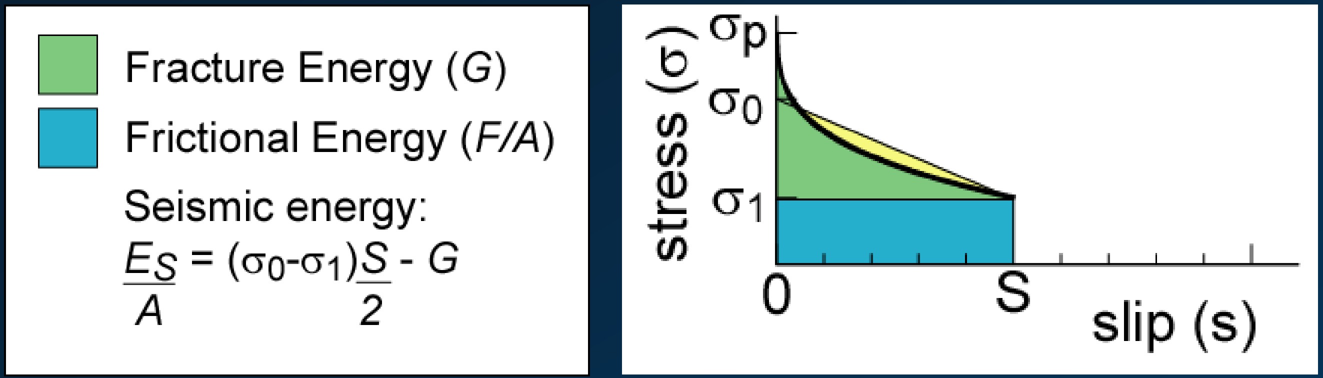




Are the measurements good enough to resolve these models?

Rachel Abercrombie: SCEC 2024

Seismic Estimates of G' – at each point, or an "average" point



$G' = \Delta\sigma/2 * s$ – Energy/ Area

Measure:
Stress drop $\Delta\sigma$
Slip s
Seismic radiated Energy E_S
Rupture area $A \sim r^2$

Either – use dynamic and kinematic modeling to measure spatially varying parameters in a finite fault inversion (Elisa will talk more about this)

Higher Quality and Quantity of data: Larger earthquakes

OR

Use average values for whole rupture
Assume simple (circular) source models

Lower Quality and Quantity of data: Smaller earthquakes

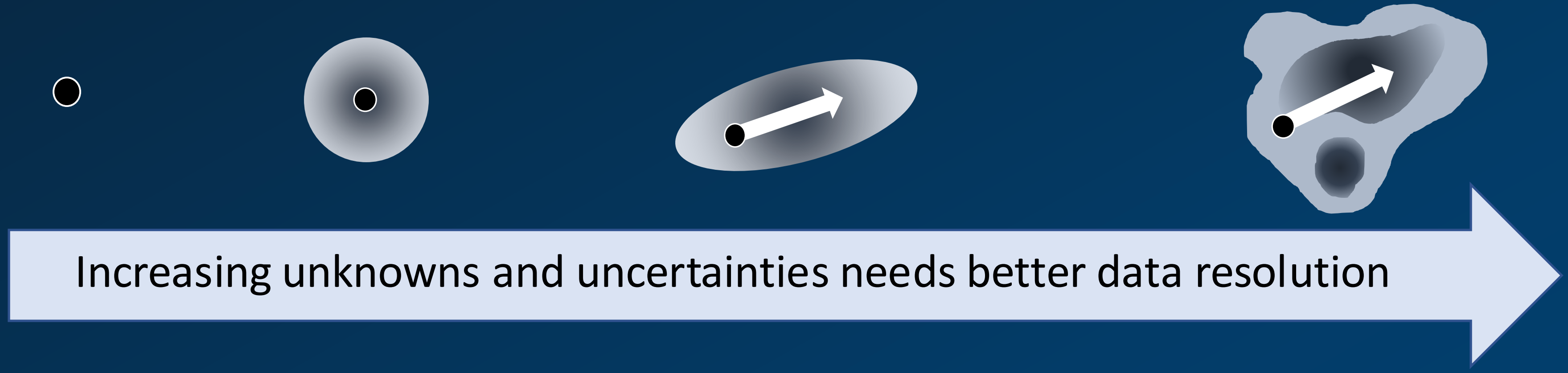
How to Estimate Stress Drop

Point

Circle:
Radius

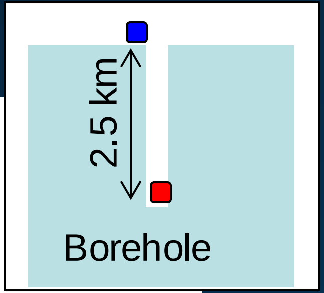
Line/Ellipse:
length, direction

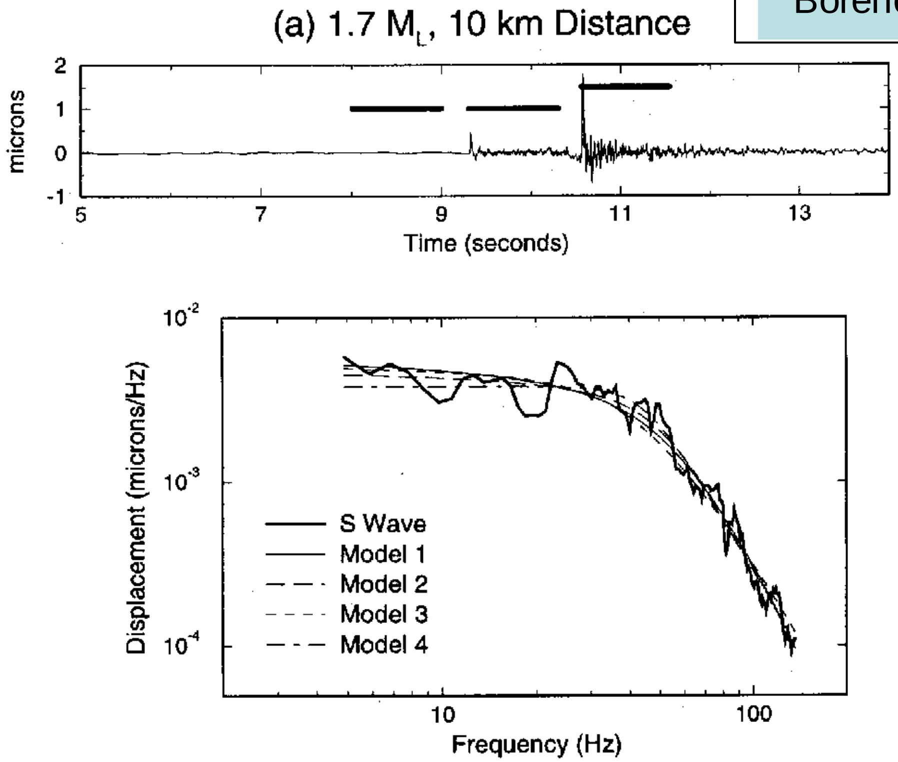
Complex Slip distribution
Asymmetrical, directional rupture



Earthquake Frequency Spectra are Magnitude dependent

Small Earthquake





Recording in Cajon Pass Borehole

Large Earthquake: M6.9

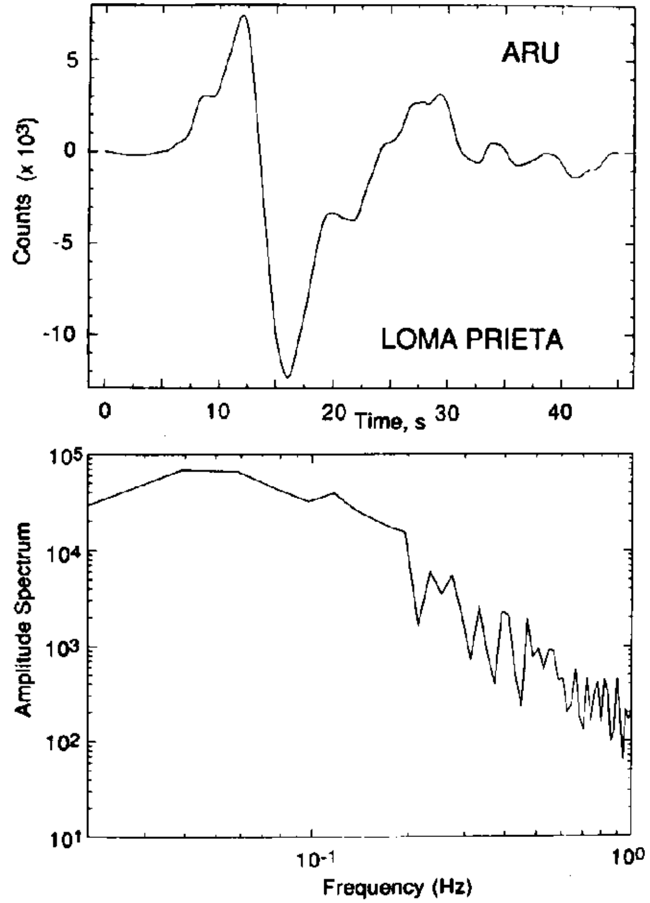
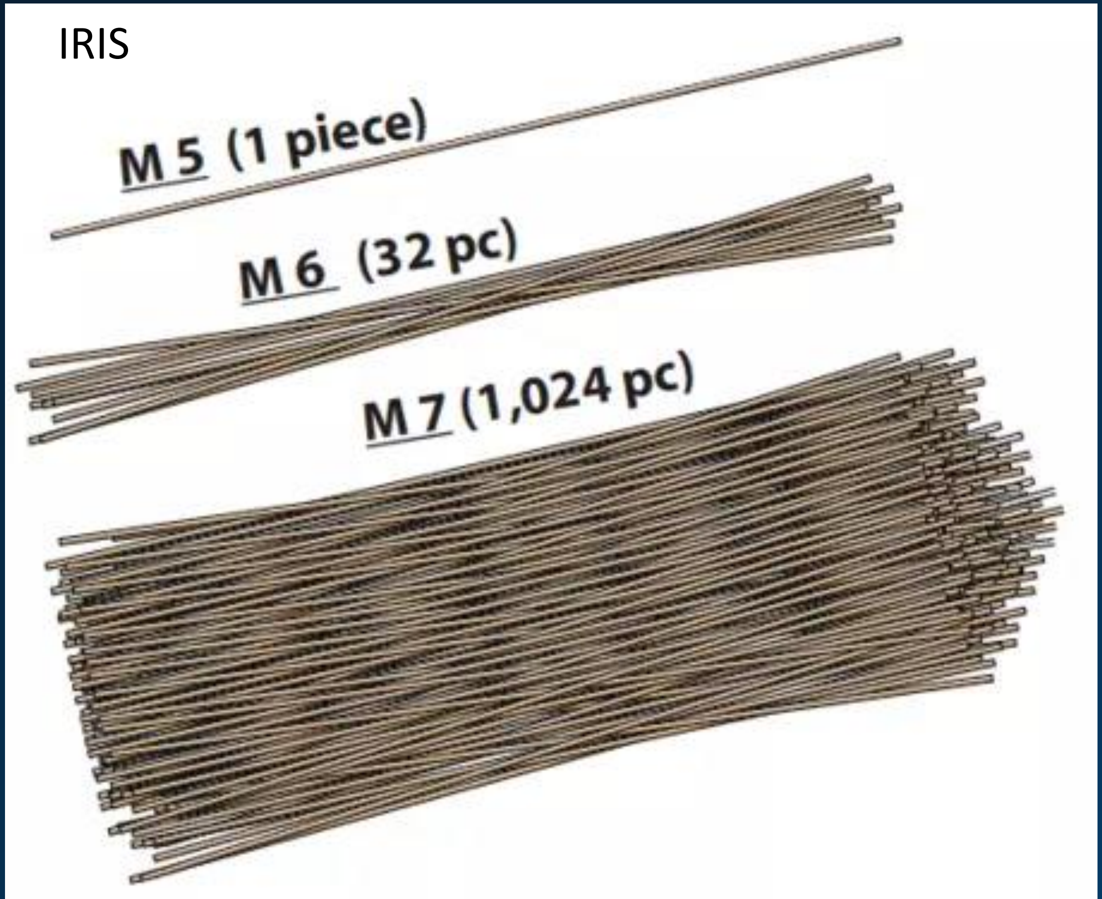


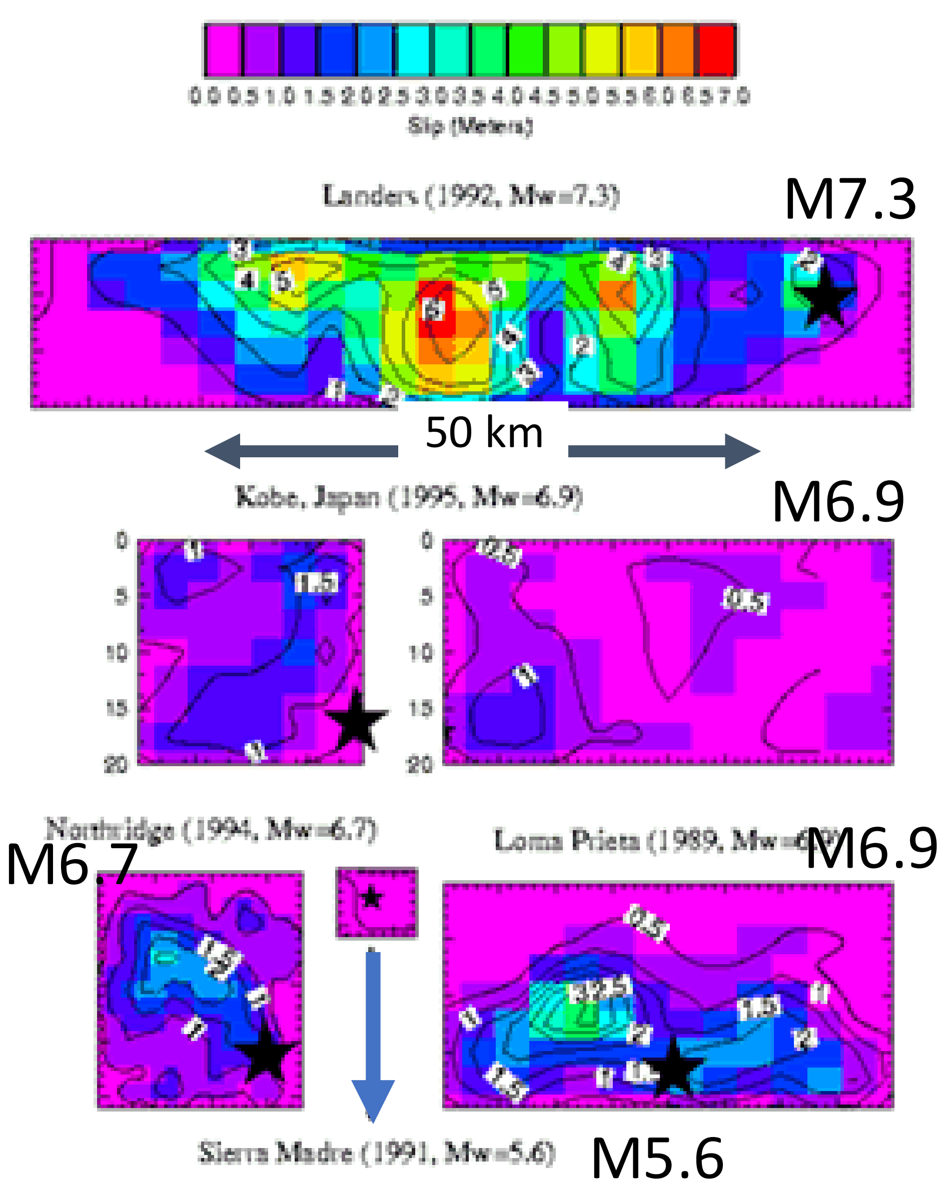
FIGURE 9.14 (Top) *P*-wave arrival from the Loma Prieta earthquake recorded at the IRIS station ARU located in Russia. (Bottom) The spectrum of the *P*-wave arrival.

Rachel Abercrombie: SCEC 2024

Brief aside:
How big are earthquakes?



David Wald

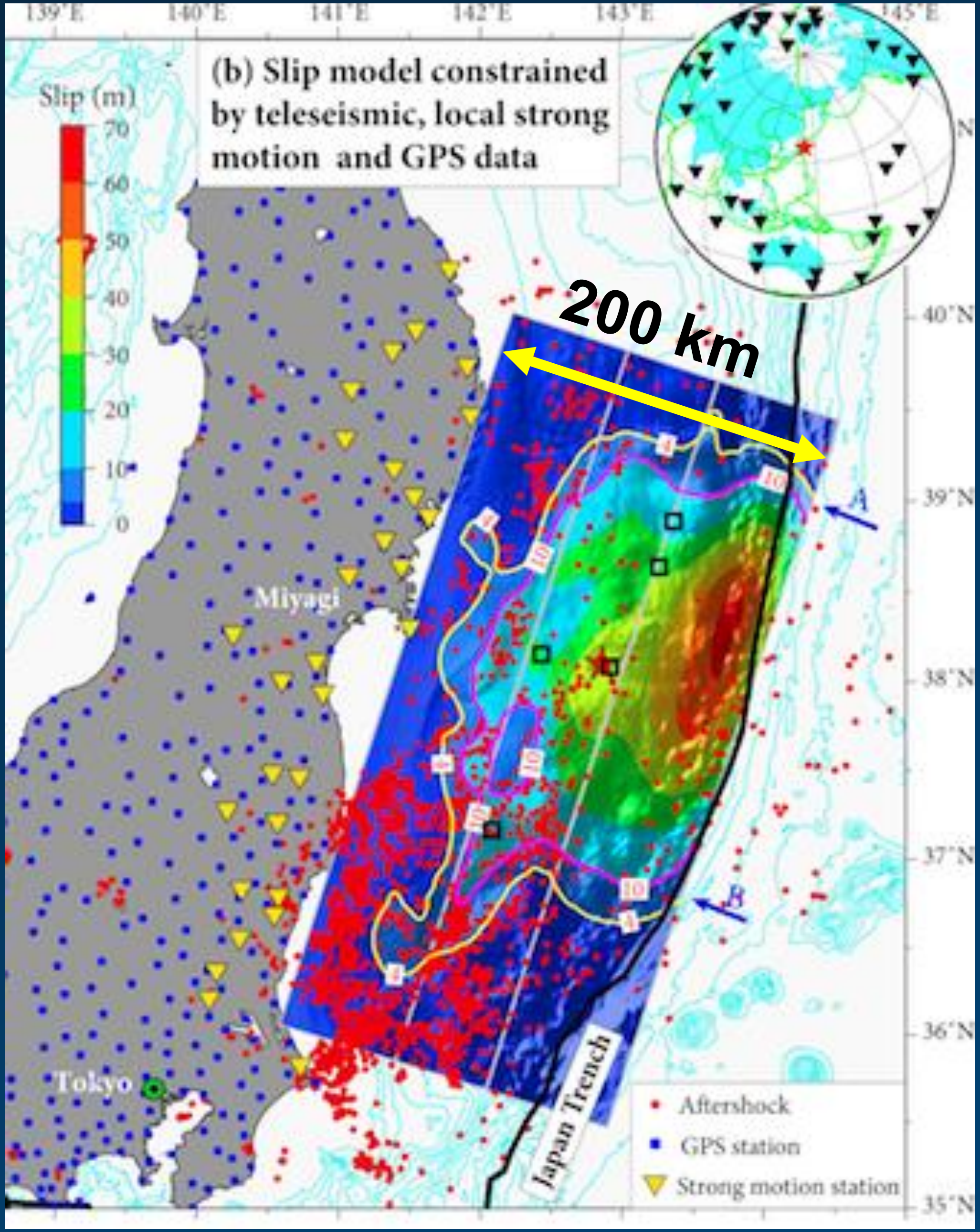


1 M increase = 30 x Energy release

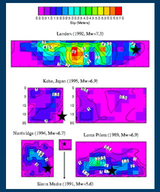
Rachel Abercrombie: SCEC 2024

Largest Earthquakes at Subduction Zones: a M9 is BIG!

2011 M9 Tohoku, Japan,
Shao *et al.*, 2011



Same scale, but slip 10 x less

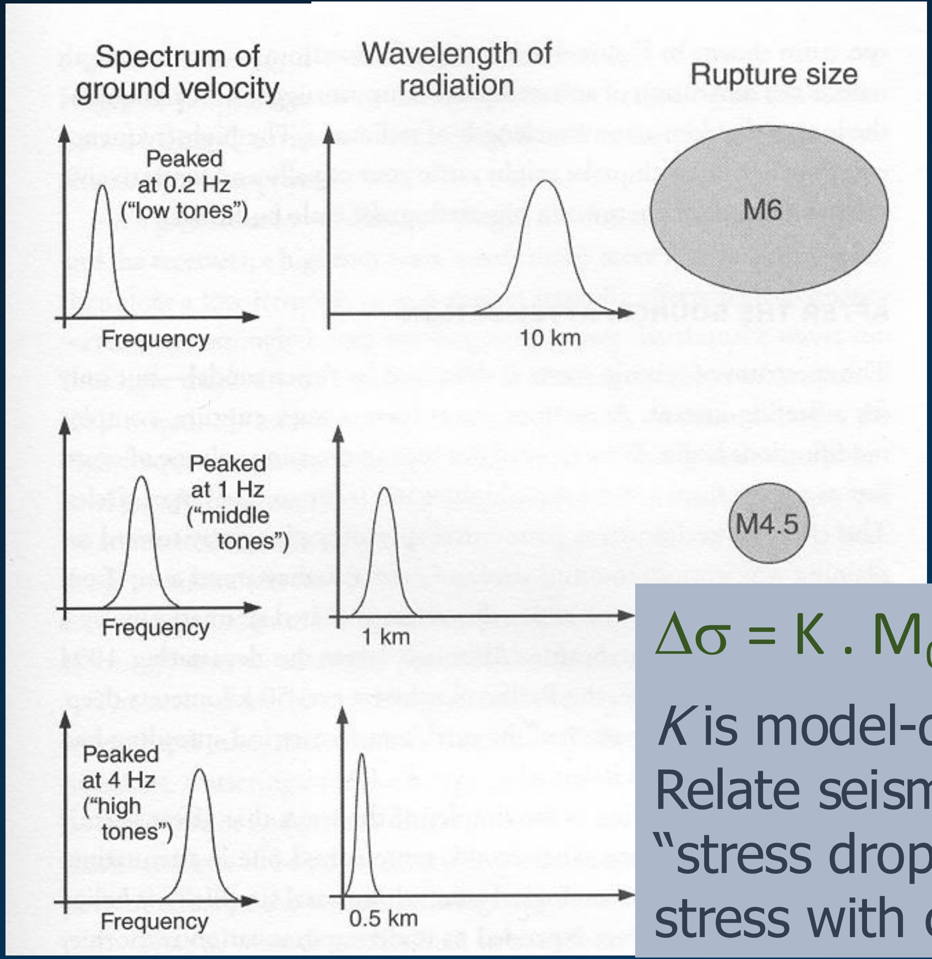


Measurements from Frequency of Seismic Waves

1960s and 1970s Aki, Brune, Kanamori, Madariaga etc.

(pre digital recording and big computers..)

Source spectrum has simple shape:



Earthshaking Science, Sue Hough, 2002

Peak in velocity spectrum

= corner frequency (fc) in displacement

~ 1/ **source dimension**

Brune Model:

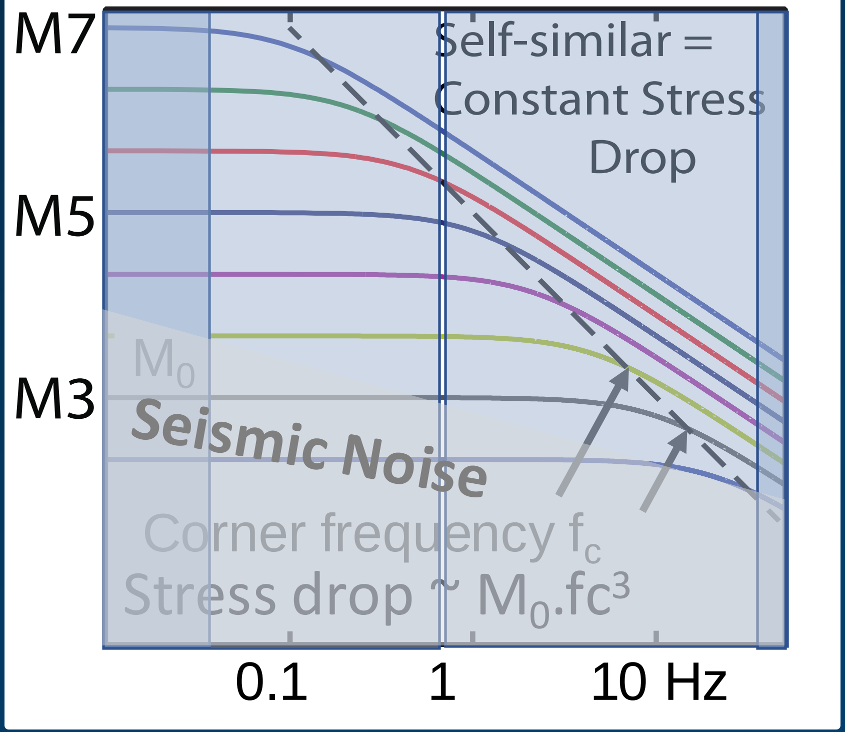
$n=2$

$$M_0(f) = \frac{M_0}{1 + \left(\frac{f}{f_c}\right)^n}$$

$\Delta\sigma = K . M_0 . f_c^3$

K is model-dependent. Relate seismological "stress drops" to real stress with care!

Model Earthquake Spectra



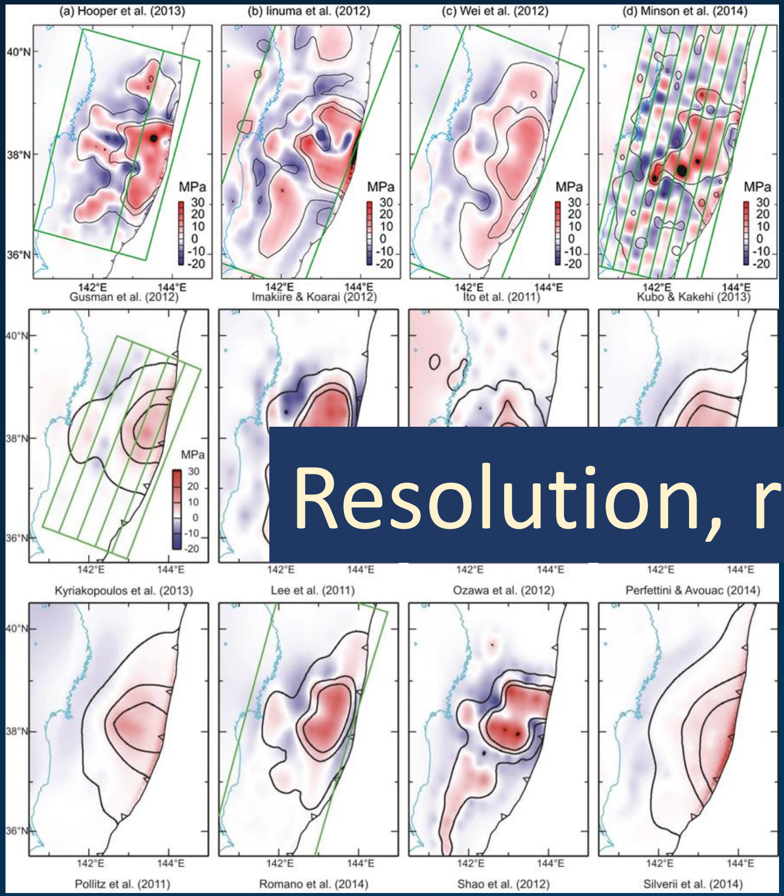
Seismic Moment: M_0 = rigidity × slip × area = Long period level

"Stress Drop" ~ strain ~ slip / $\sqrt{\text{area}}$

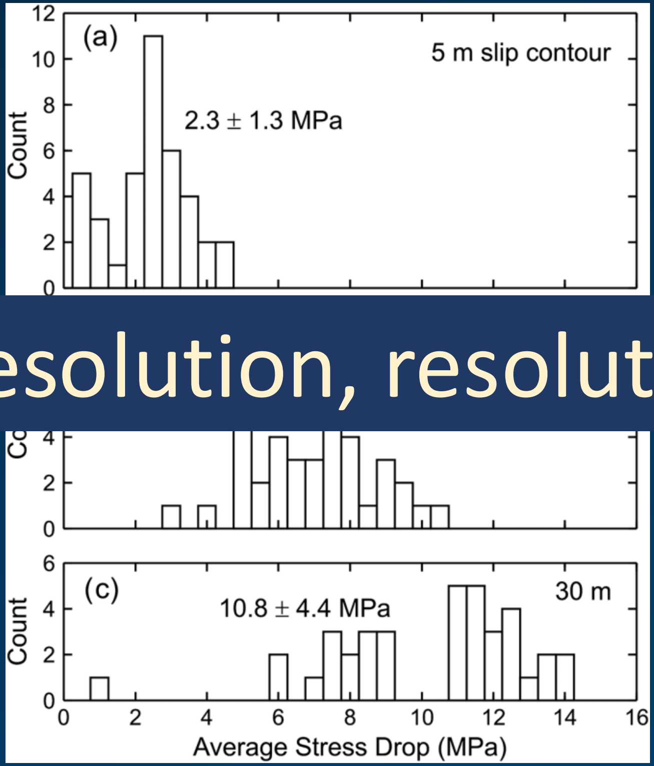
How to Measure Earthquake Stress Drop and Radiated Energy? How many numbers do you need to characterize and compare earthquakes?

Large Earthquakes: Finite Fault inversions, Teleseismic v. regional data

Stress Drop: Brown *et al* 2015 Tohoku-Oki M9

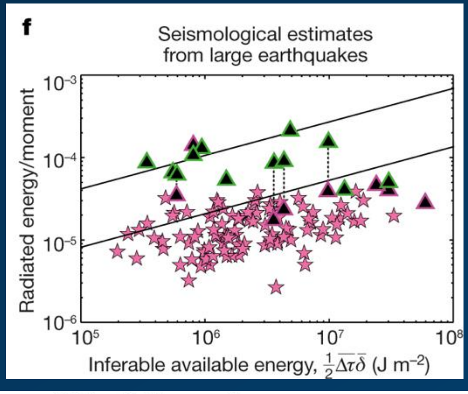


Calculate average for each of 40 models



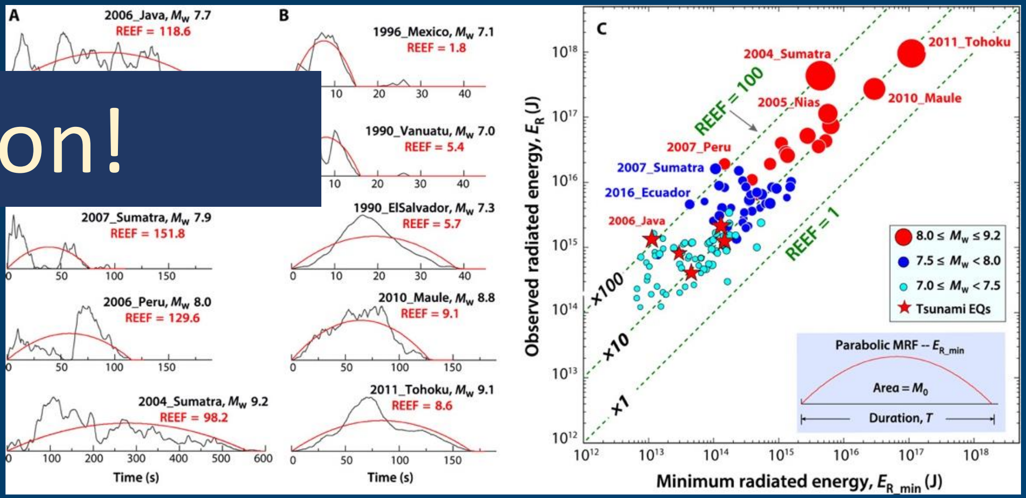
Resolution, resolution, resolution!

Radiated Energy



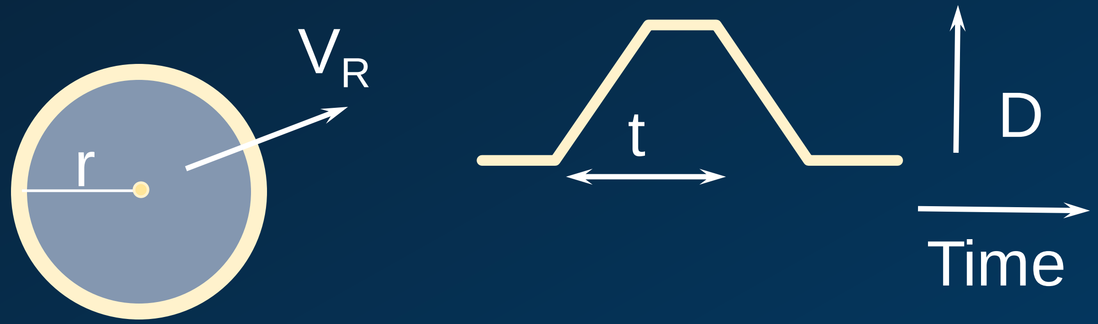
Lambert et al. 2021

Ye et al., 2018

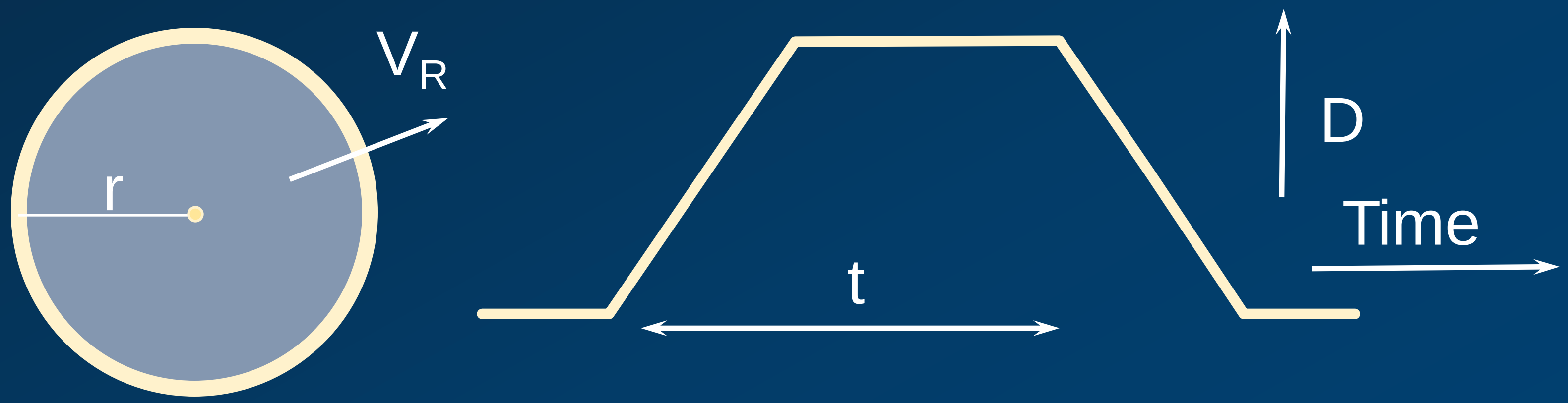


Information from Seismic Waves

Small Earthquake: small time, small area, small slip



Large Earthquake: large time, large area, large slip



$$r = V_R \times t$$

What is Rupture Velocity (V_R)?

Rachel Abercrombie: SCEC 2024

Early Work

Thatcher and Hanks, 1973
Source Parameters of Southern California Earthquakes

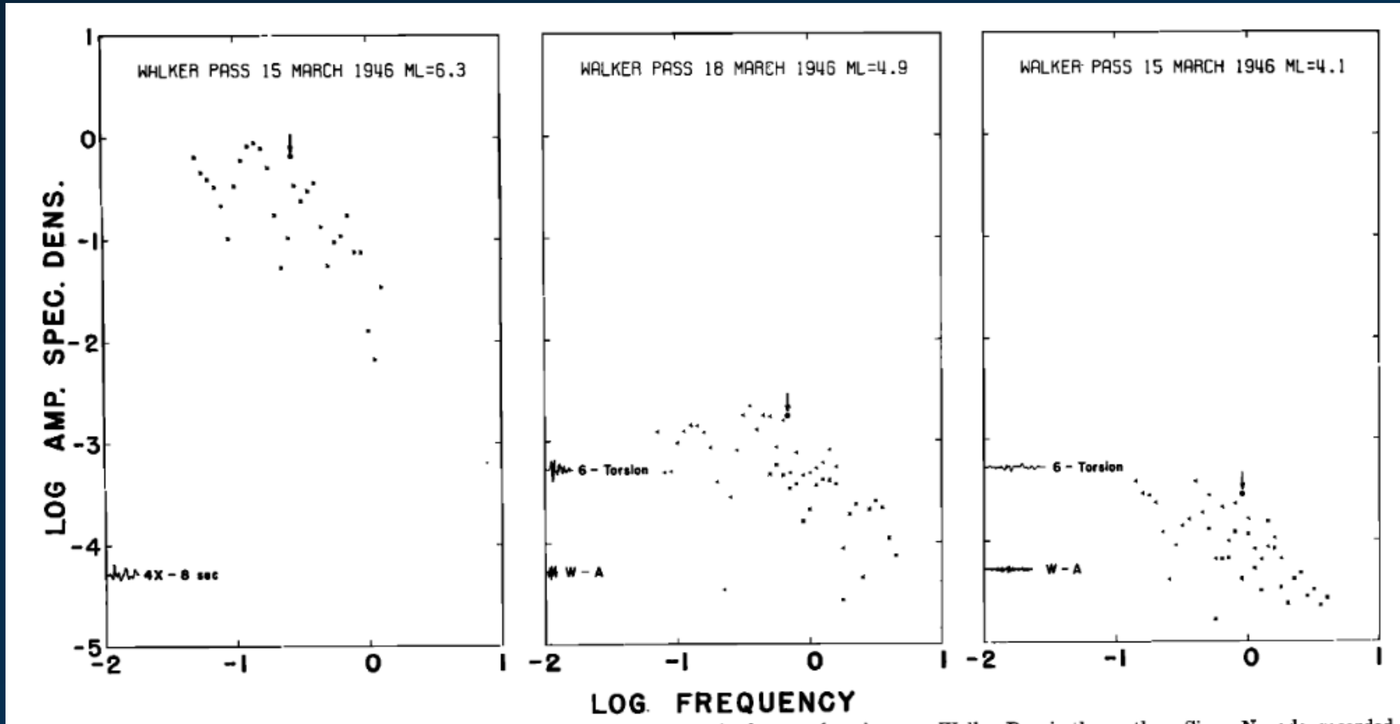
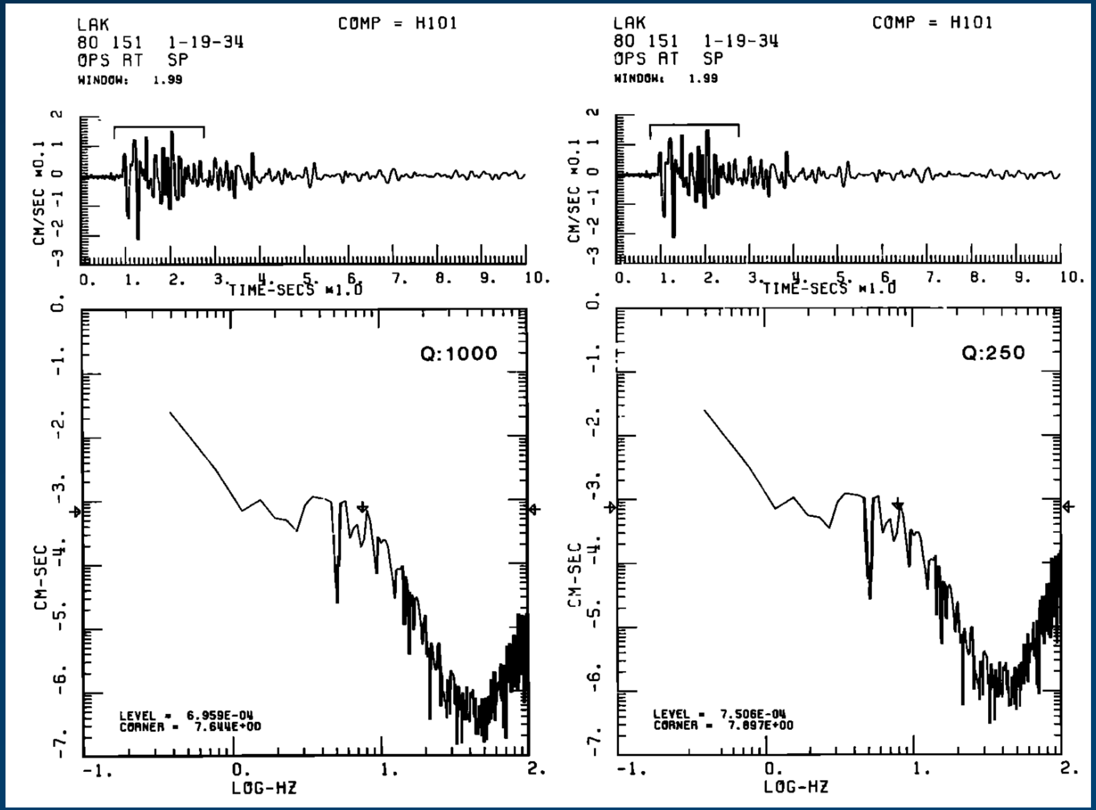


Fig. 4. Seismograms and spectra from three earthquakes, all in approximately the same location near Walker Pass in the southern Sierra Nevada, recorded at Pasadena (Δ ≃ 170 km). Dot with arrow shows intersection point of long-period level and high-frequency asymptote.

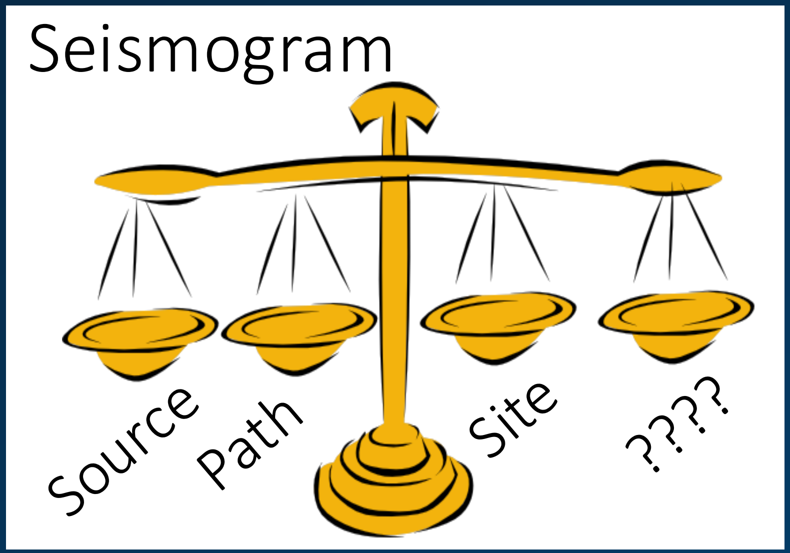
Archuleta et al, 1982
Mammoth Lakes earthquakes

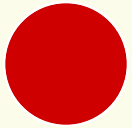
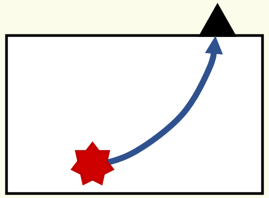
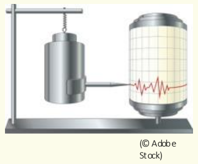


Rachel Abercrombie: SCEC 2024

Estimating Source Parameters

First Problem: how separate source and path in recorded seismograms



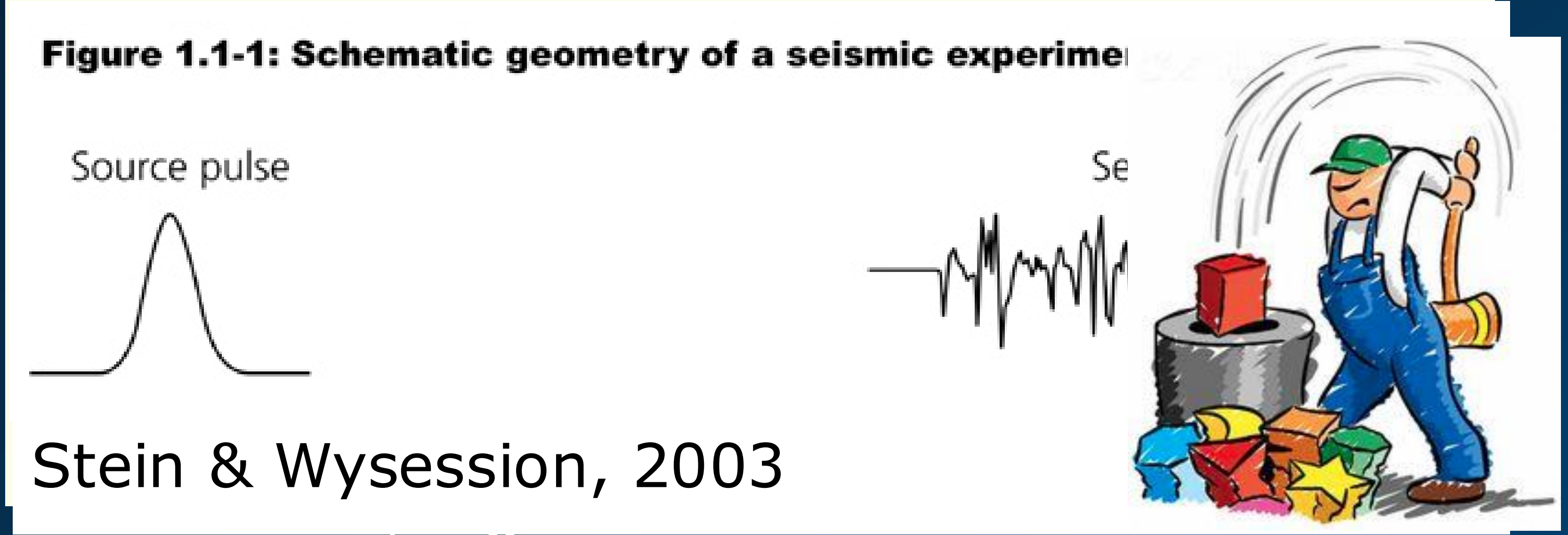
	Source	Path + Site	Instrument
Simple Models			
Circular source		1D (or constant) attenuation structure 	 Known – hopefully

Seismogram (X):

$X(t) = S(t) * G(t) * I(t)$

Frequency spectrum:

$X(f) = S(f) \times G(f) \times I(f)$



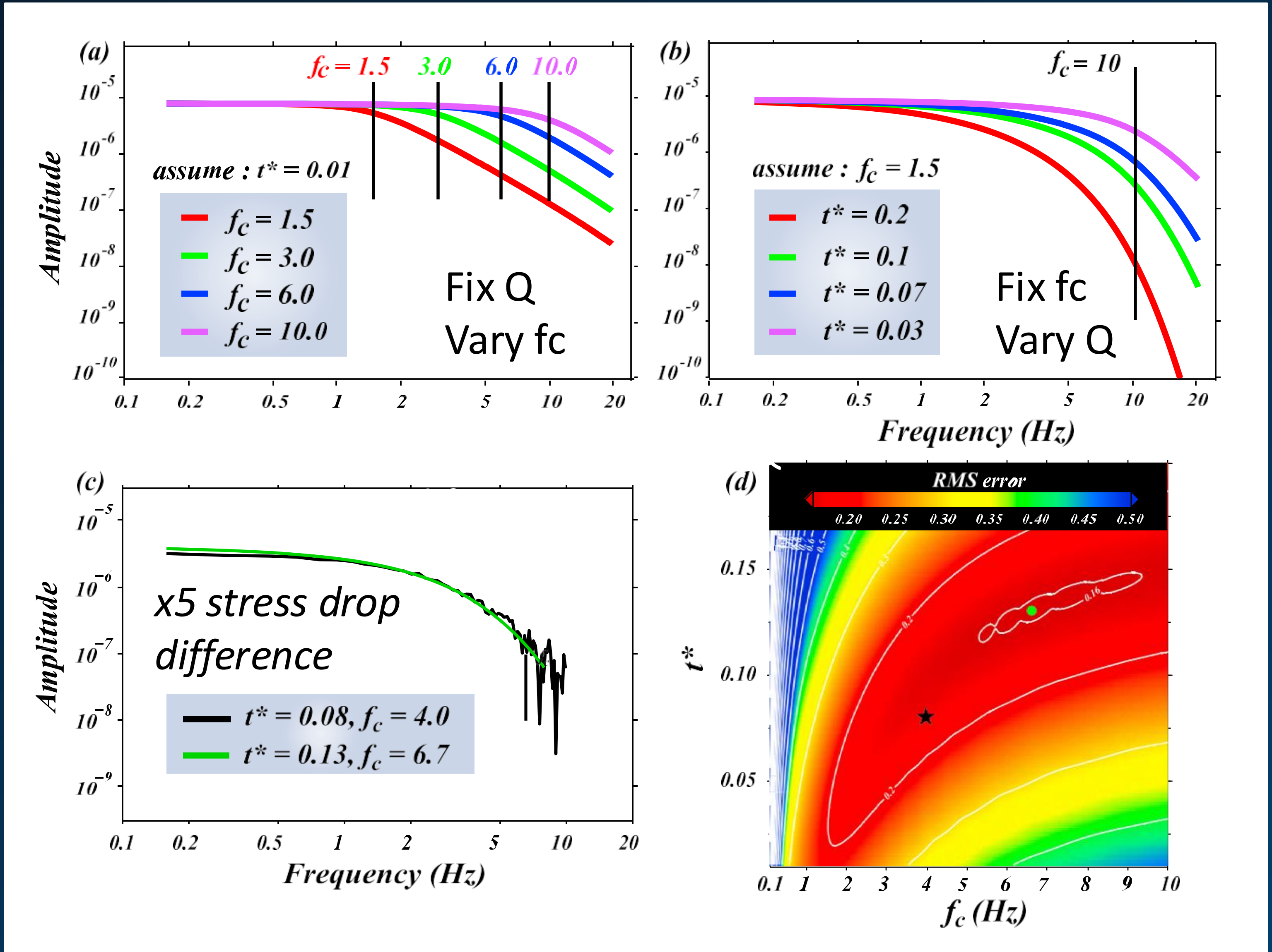
Model attenuation, or use small earthquakes as Empirical Green's functions

In practice: Trade-offs.

Hard to resolve with limited frequency range data - frequency OR time domain

See review : Abercrombie (2021) *Phil Trans. Royal Soc.*

Extracting Source from Seismograms: Ko *et al., JGR* 2012: *fc and Q* Modelling ambiguity



$t^*=t/Q$

Large trade-offs in limited frequency range

$$M_0(f) = \frac{Ce^{-\pi f t/Q}}{1+\left(\frac{f}{f_c}\right)^2}$$

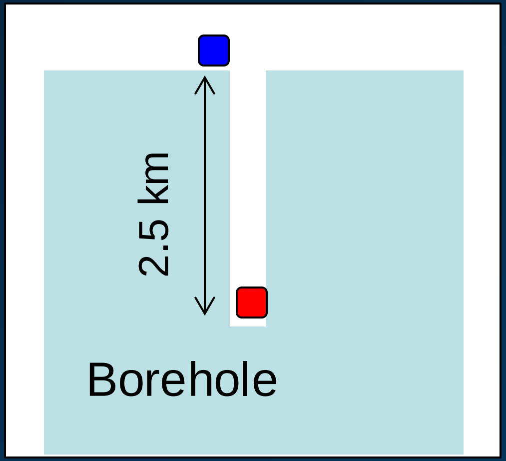
Can improve using cluster or stacking & joint inversion analysis…

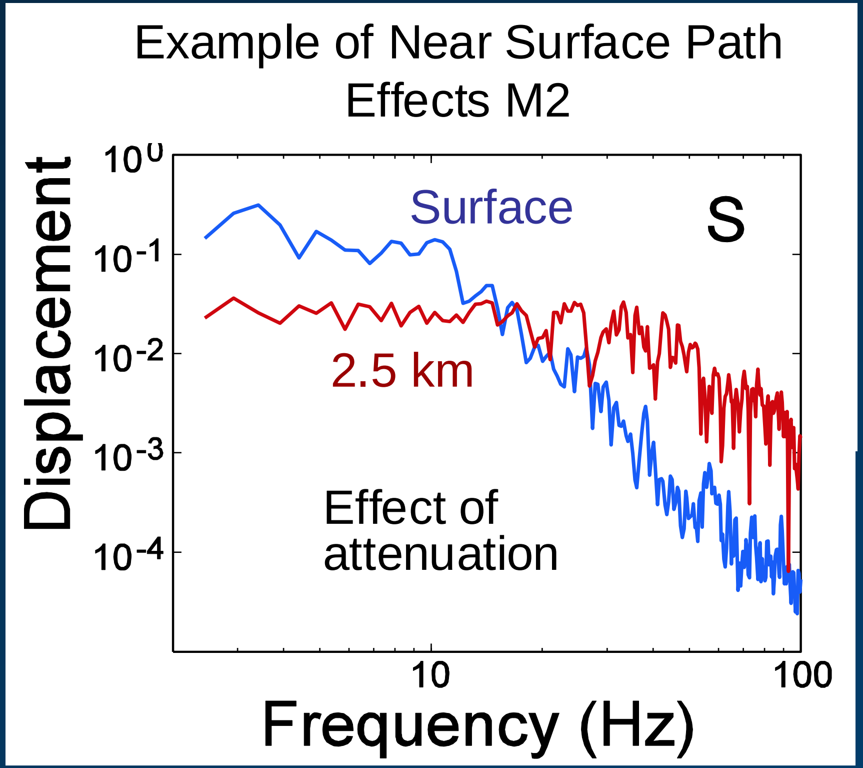
Site Effects = Shallow (10 m? 100 m? 1 km?)

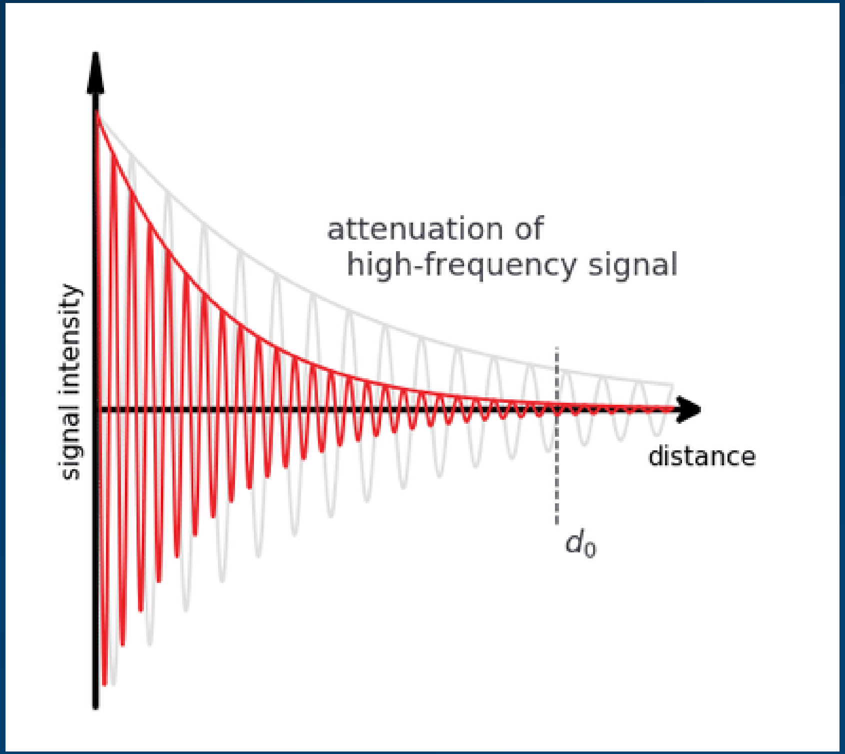
Site Effects ~ attenuation and amplification near surface. Can be as large as rest of path. Not dependent on station distance

Typically Model path and site by adding exponential attenuation to circular Source model

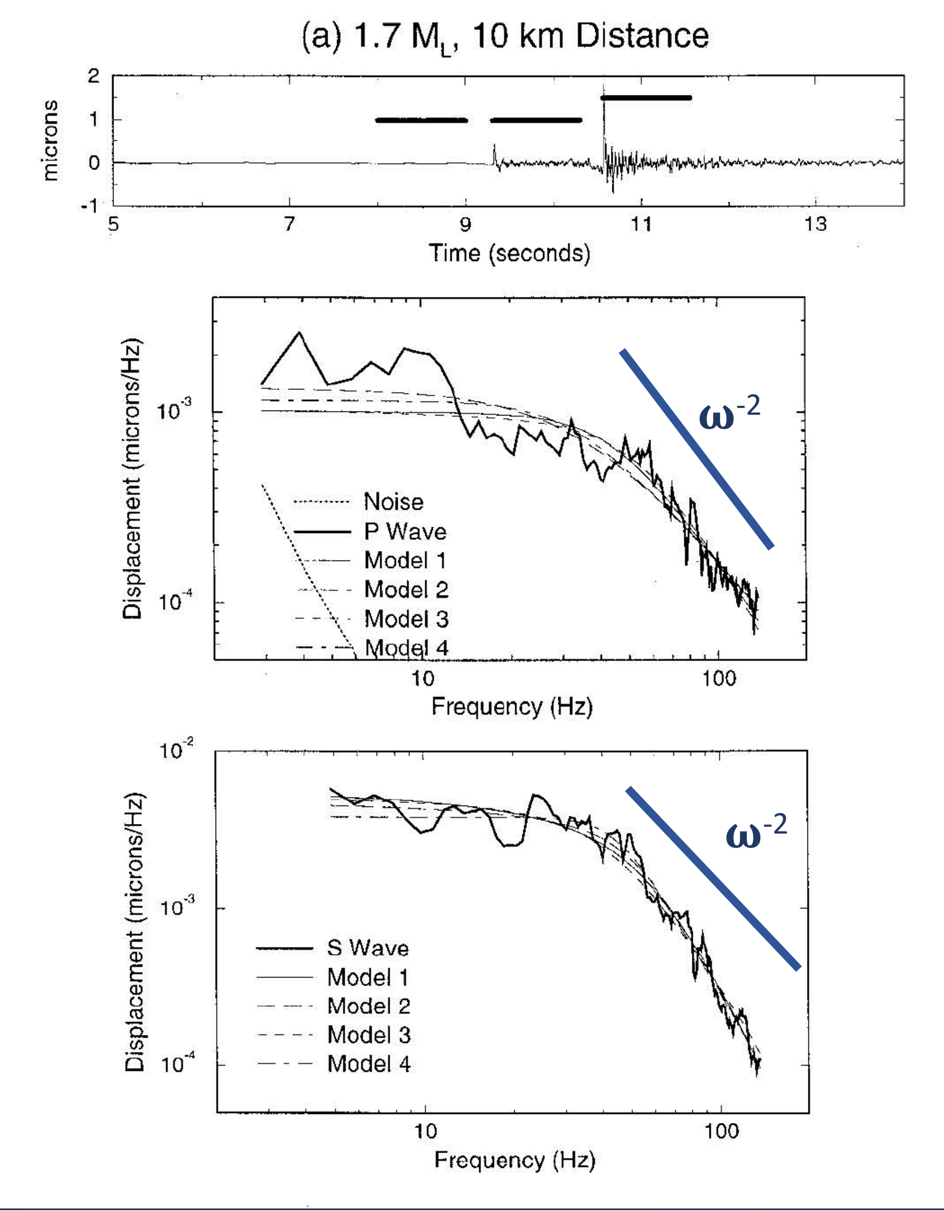
Abercrombie, JGR 1995

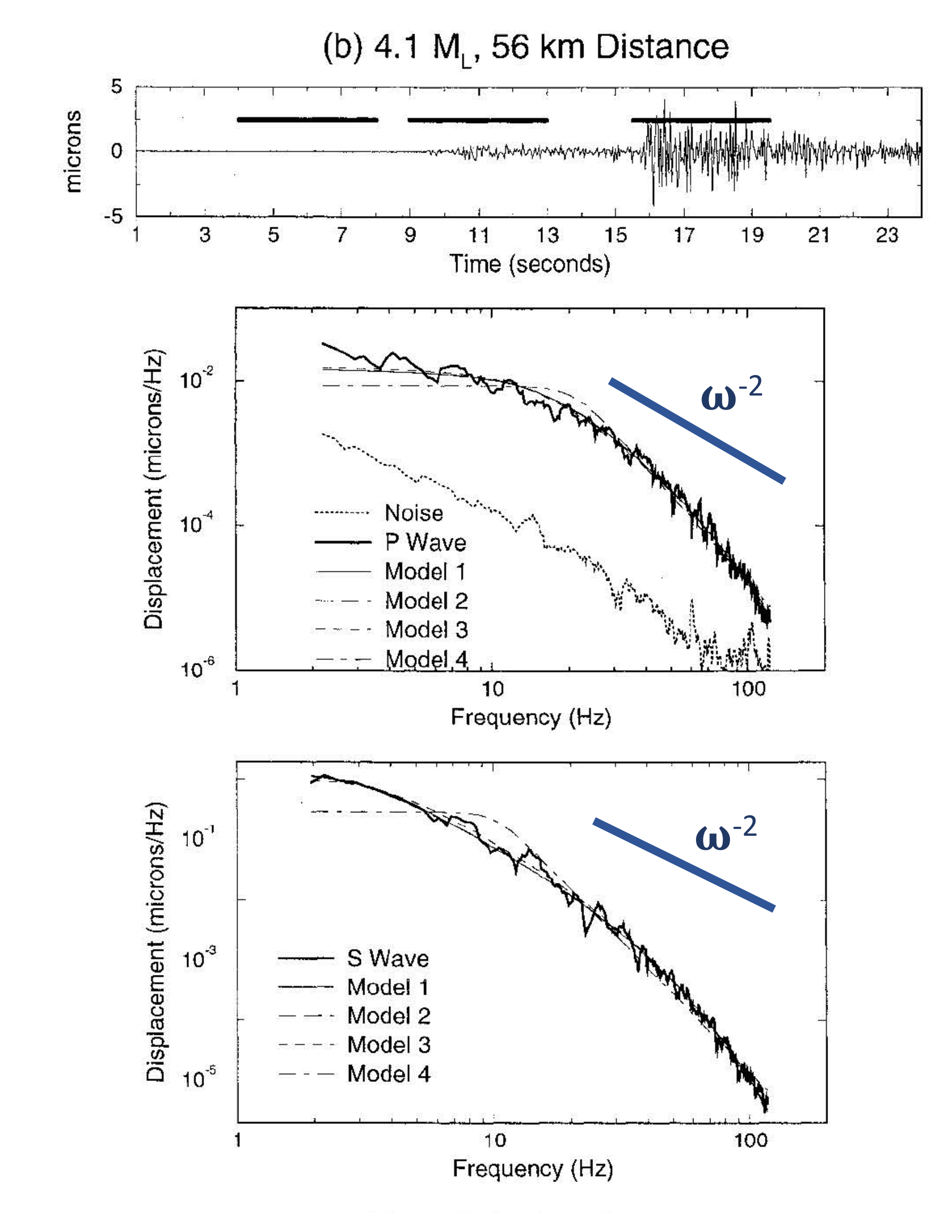






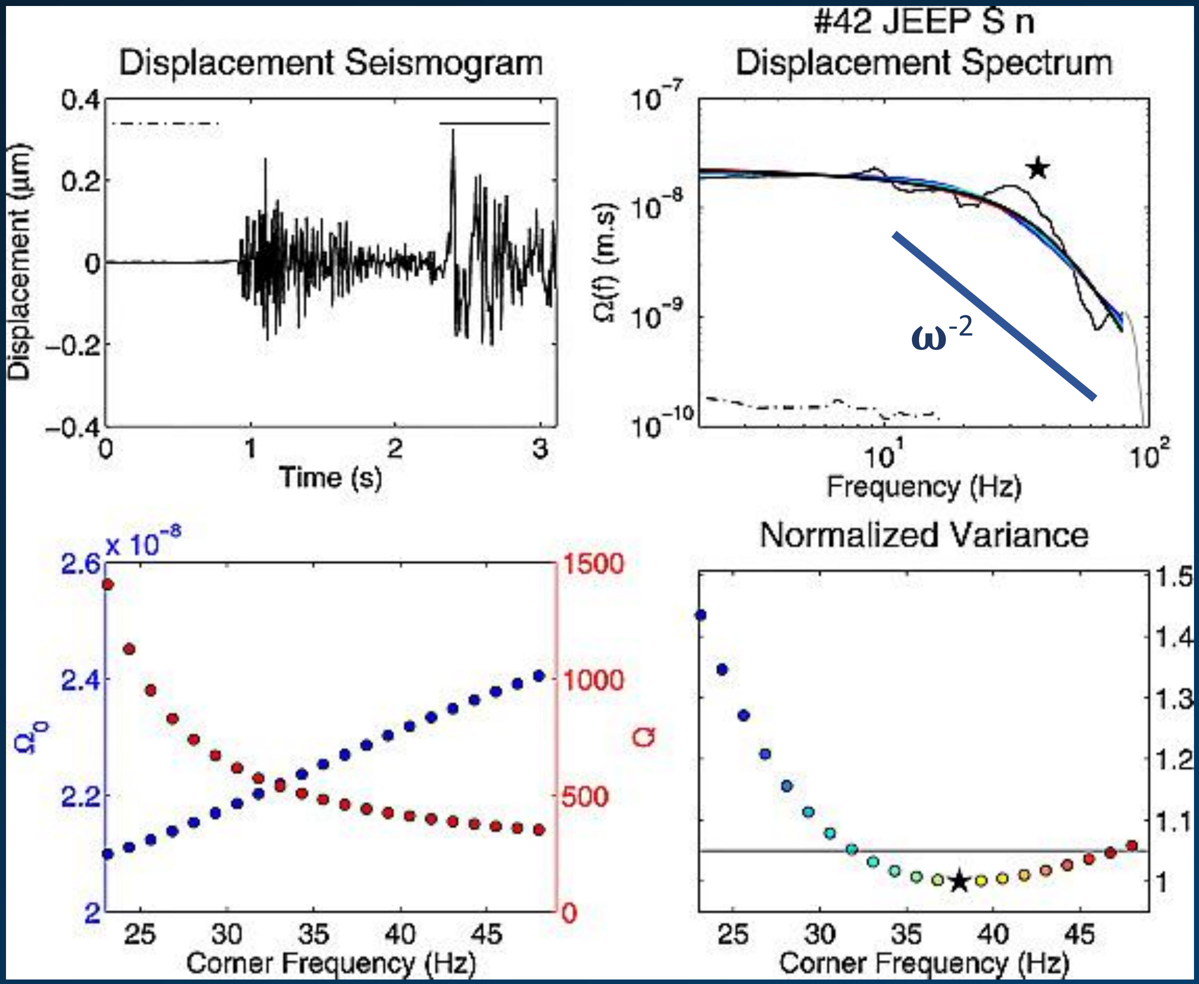
Abercrombie (1995) Fitting Spectra at 2.5 km depth

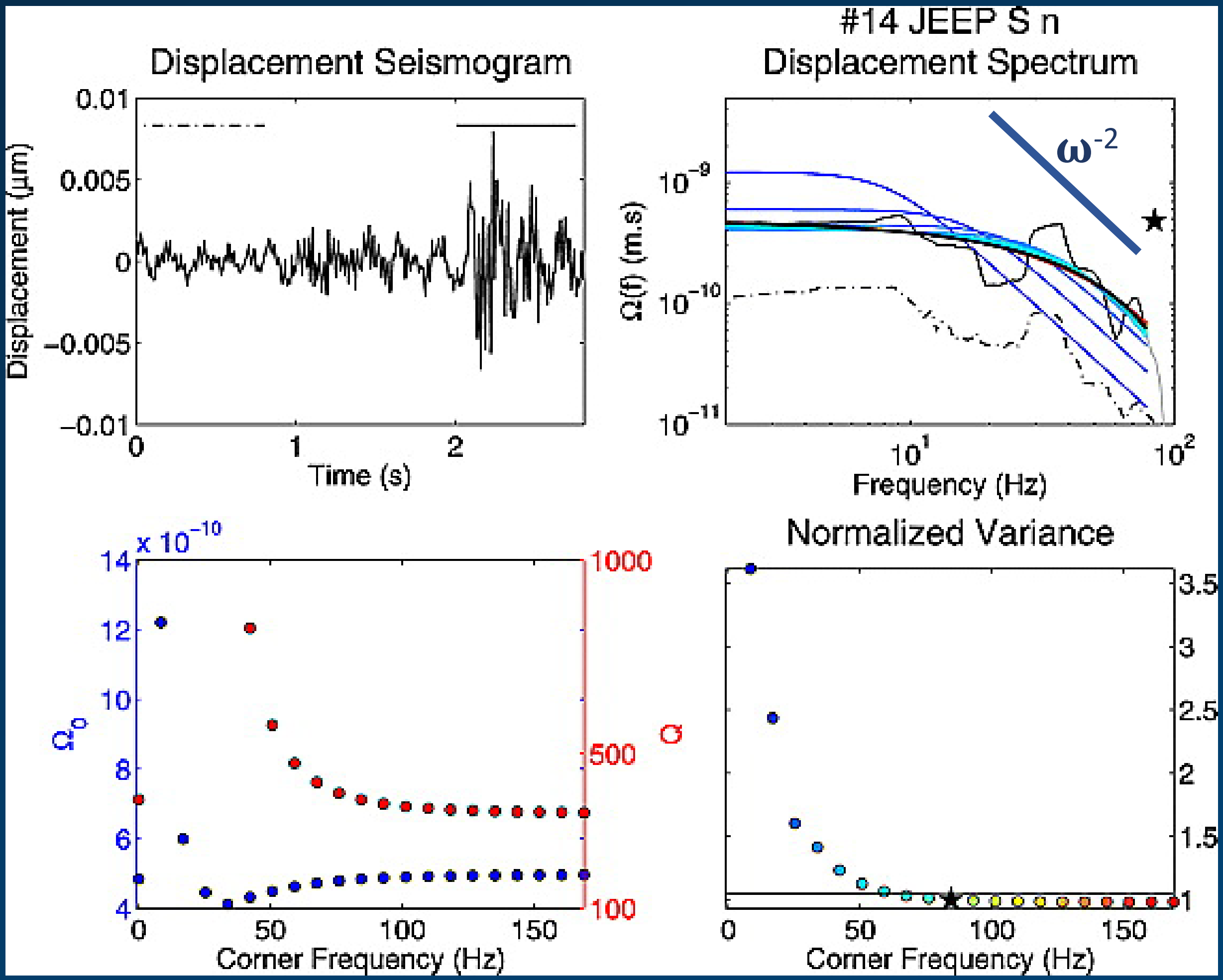




Rachel Abercrombie: SCEC 2024

Viegas et al. (2010) Fitting Spectra of NY eqs





How to calculate and fit a spectrum, then to Stress Drop

Rachel Abercrombie: SCEC 2024

Calculate Spectrum

- Time window – affects lowest f
- Sampling rate – limits highest f
- Tapering (cosine, multi-taper)
- Frequency range above noise (What is good signal/noise ratio?)

Fit Spectrum

- Source v. Path v. Site and geometrical spreading
- Source model
- Constraints / combined Inversion scheme
- Range of good fits? Consider uncertainties in model choices, as well as fit of chosen model.

Calculate Stress Drop

- Moment estimate
- Duration/corner frequency > length
- Length > area
- Moment and area > stress drop

3 main equations

$$M_0(f) = \frac{Ce^{-\pi f t/Q}}{1+\left(\frac{f}{f_c}\right)^2}$$

$$r = \frac{k\beta}{f_c}$$

$$\Delta\sigma = \frac{7}{16}\frac{M_0}{r^3}$$