SCEC Annual meeting
Palm Springs | 8 September 2024
Stress drop workshop

Earthquake source parameters from probabilistic inversion of displacement spectra using a single-station approach

Mariano Supino

INGV

Content

METHOD | ESTIMATION OF SOURCE PARAMETERS

Global exploration of the model space

Joint Probability Density Function of source parameters

Marginal PDFs

FROM RAW SIGNAL TO SOURCE PARAMETERS

SINGLE STATION APPROACH
MAIN BENEFITS AND LIMITATIONS

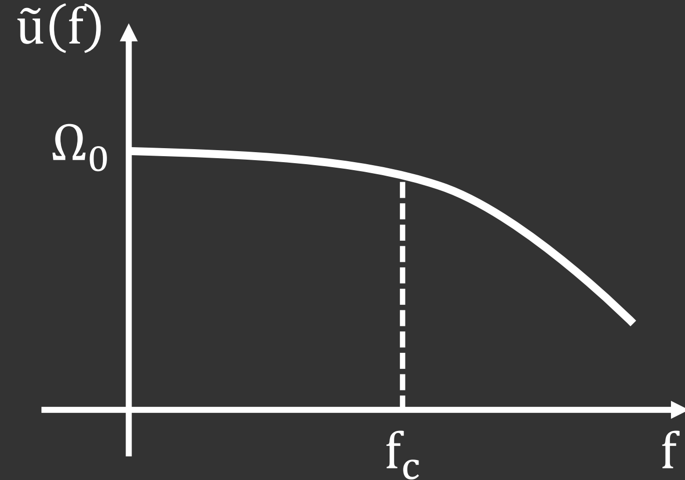
APPLICATIONS

INVERSE METHOD

Observed data and model predictions

OBSERVED DATA $\boldsymbol{d}$

Far field displacement spectrum $\tilde{u}(f)$



MODEL PREDICTIONS

Forward operator

$$g(\boldsymbol{m}; f) = \tilde{S}(M_0, f_C, \gamma; f) \cdot \tilde{G}(Q; f) \cdot \widetilde{H}(f)$$

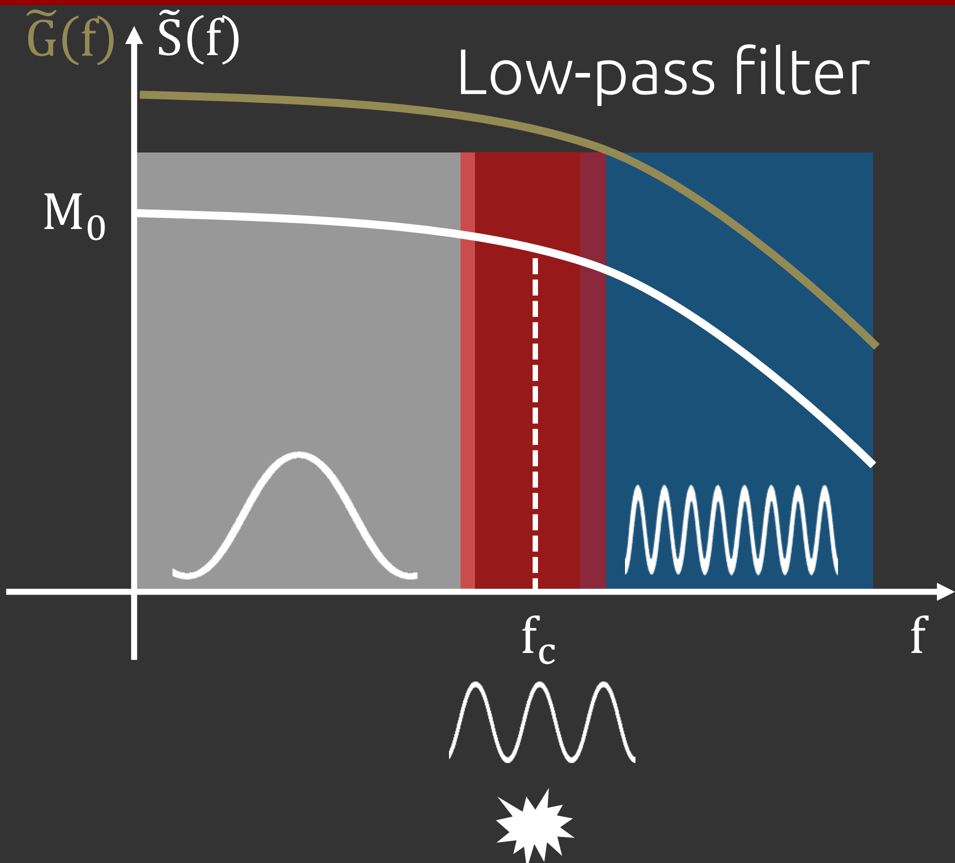
SOURCE SPECTRAL MODEL | Brune

PROPAGATION MODEL | TGF

SITE FACTOR

Observed data and model predictions

Expected displacement spectrum



M_0 | Seismic moment ➡ Magnitude

f_c | Corner frequency ➡ Rupture dimension

γ | Decay exponent ➡ Rupture interactions

Q' | Anelastic attenuation Q-factor + site-dependent attenuation term k_0

SOURCE

Brune spectral model

$$\tilde{S}(M_0, f_c, \gamma) = \frac{M_0}{1 + \left(\frac{f}{f_c}\right)^{\gamma}}$$

PROPAGATION

Theoretical Green's function

$$\tilde{G}(Q') = A_r \cdot \exp(-\pi f T_r \cdot Q')$$

Travel time (T_r)

Q and k_0 (Q')

Probability density function (PDF) estimation

Tarantola's approach : Conjunction of states of information

$$\sigma\,(\boldsymbol{m},\boldsymbol{d}) = k \cdot \rho_M(\boldsymbol{m})\rho_D(\boldsymbol{d}) \cdot \theta(\boldsymbol{d}|\boldsymbol{m})$$

CONJUNCTION OF TWO STATES OF INFORMATION

Tarantola, 2005

A priori on model and data space

Physical correlation between **d** and **m** : FORWARD OPERATOR $\boldsymbol{g}(\boldsymbol{m})$

$$\sigma_M(\boldsymbol{m}) = \int_D \sigma\,(\boldsymbol{m},\boldsymbol{d})\,d\boldsymbol{d}$$

SOLUTION OF THE INVERSE PROBLEM

Joint a-posteriori PDF over the model space

Integration of $\sigma_M(\boldsymbol{m})$

PARAMETER SOLUTION	UNCERTAINTY
Mean	Variance

A-posteriori pdf estimation
Modelization and data uncertainties Gaussian assumption

$$\rho_D(\boldsymbol{d}) = k' exp\left(-\frac{1}{2}(\boldsymbol{d} - \boldsymbol{d}_{obs})^T \boldsymbol{C}_D^{-1}(\boldsymbol{d} - \boldsymbol{d}_{obs})\right)$$

$$\theta(\boldsymbol{d}|\boldsymbol{m}) = k'' exp\left(-\frac{1}{2}(\boldsymbol{g}(\boldsymbol{m}) - \boldsymbol{d})^T \boldsymbol{C}_T^{-1}(\boldsymbol{g}(\boldsymbol{m}) - \boldsymbol{d})\right)$$

GAUSSIAN ASSUMPTION

Analytical solution for σ_M

$$\sigma_M(\boldsymbol{m}) \propto exp\left(-\frac{1}{2}S(\boldsymbol{m})\right)$$

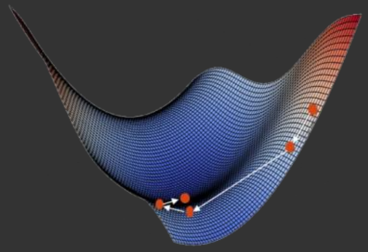
$S(\boldsymbol{m})$ cost function, squared L_2-norm between observed data $\mathbf{d}_{obs}$ and model predictions $\mathbf{g}(\mathbf{m})$

TARANTOLA, 2005

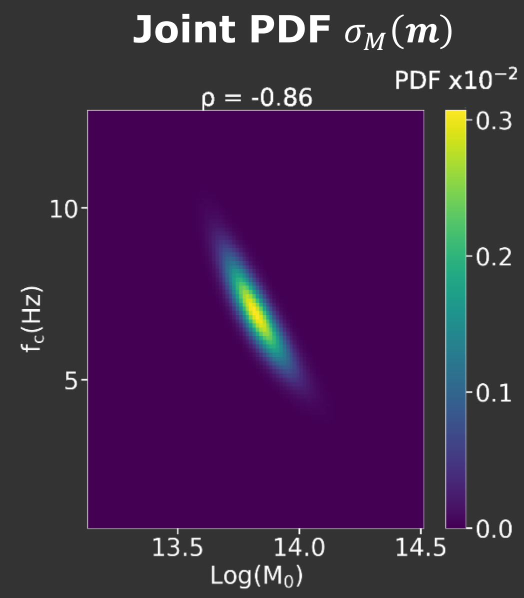
Method

SPAR | Supino et al., 2019 – GJI | Link

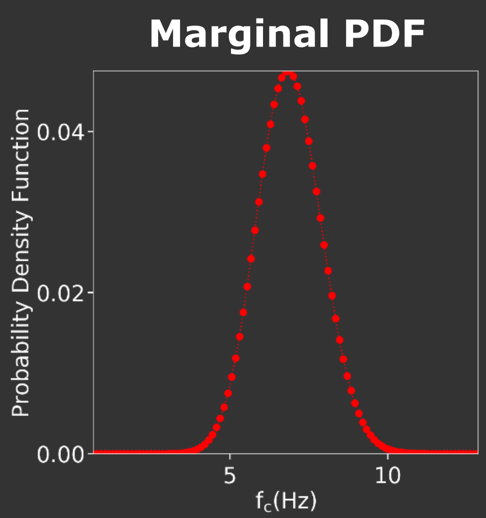
A Find the a-posteriori PDF $\sigma_M(\boldsymbol{m})$ maximum $\boldsymbol{m}^*$ through MC based global optimization



B Evaluate $\sigma_M(\boldsymbol{m})$ around $\boldsymbol{m}^*$



C Evaluate marginal PDFs and quality of the solutions



Benefits of the method

GLOBAL OPTIMIZATION

Avoid local minima during the search for the best model to describe the observed data

ROBUST UNCERTAINTIES ESTIMATION

PDF solutions allow robust uncertainty estimations, accounting for between-parameter correlations and limited frequency bandwidth.

Benefits of the method

PDF SOLUTION

In most of the cases an algorithm will be able to find a scalar solution to a minimization problem.

A PDF allows to:

- Understand if the inversion is constrained, if it ***makes sense***
- Associate an uncertainty to each inverted observation (beyond the standard deviation of the solutions)

The idea is to have a vector solution with more information about the inversion than a scalar solution.

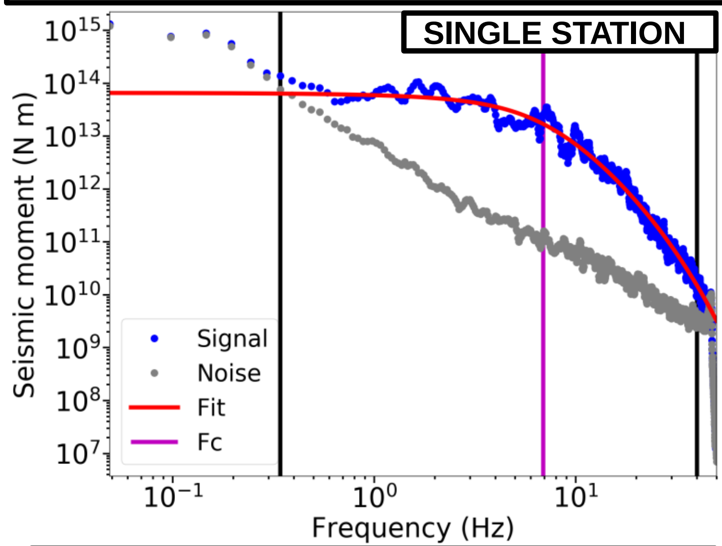
EXAMPLE

ONE EVENT INVERSION

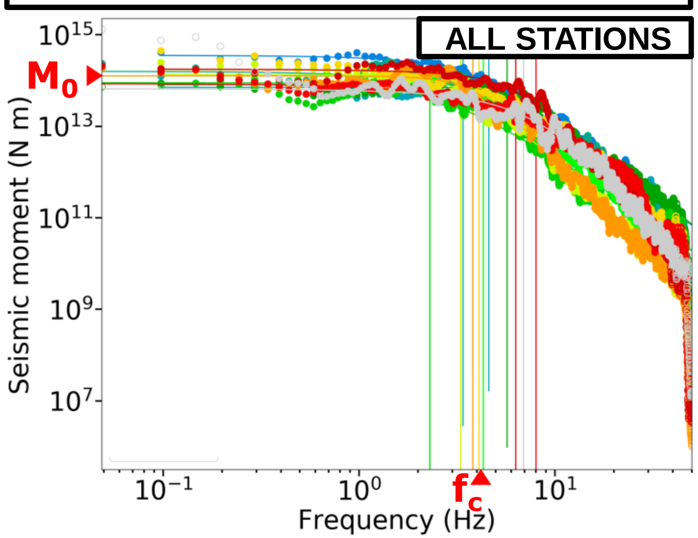
Single event inversion

All stations | Stress drop estimate

S-wave displacement spectrum



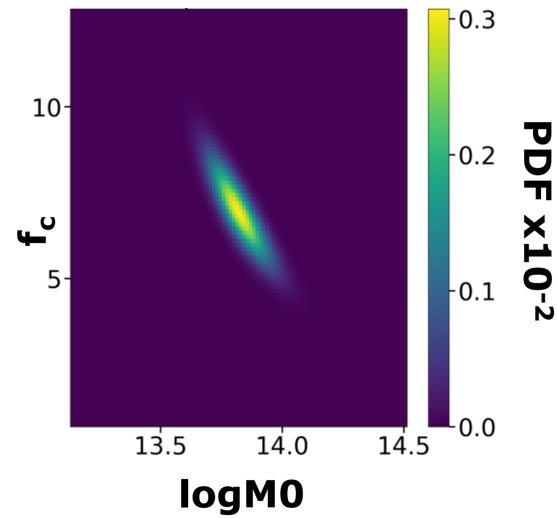
Event solution



Probability density functions

Joint PDF $\sigma_M(m)$

CORRELATION $\rho = -0.86$



$$r = \frac{k}{f_c}$$

↓ SOURCE RADIUS

$$k = k(v_R)$$

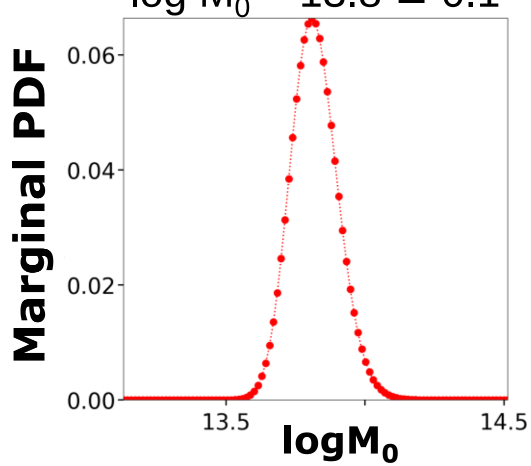
↙ SOURCE MODEL and PHASE ↓ RUPTURE VELOCITY

$$\Delta\sigma = \frac{7}{16}\frac{M_0}{r^3} = 12 \pm 6\ MPa$$

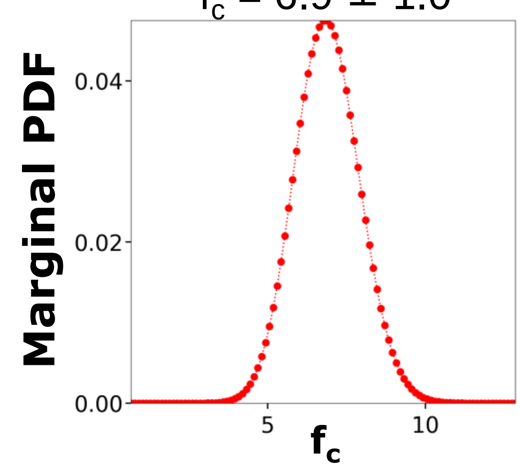
↓ STRESS DROP

↘ **k = 0.26**, Kaneko and Shearer, 2014 S-waves

log M_0 = 13.8 ± 0.1



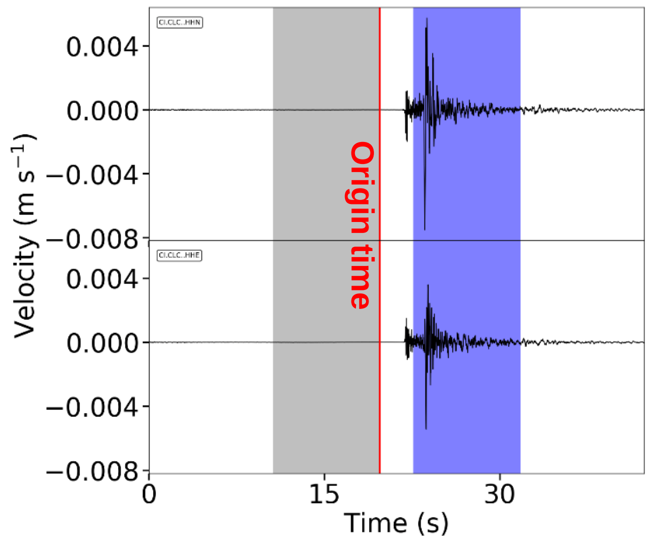
f_c = 6.9 ± 1.0

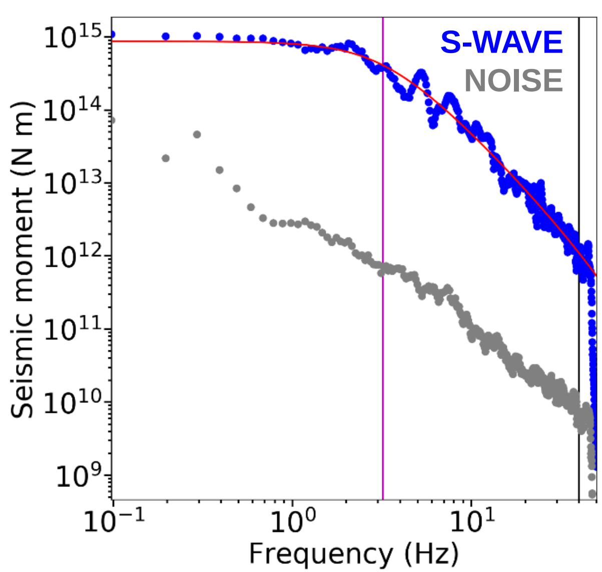


FROM RAW SIGNALS TO SOURCE PARAMETERS

From raw signals to source parameters
Phase selection | Time window

Phase selection: S- or P-wave





Time window Δt

- Fixed (e.g., 5 seconds)
- Function of magnitude and/or distance

$$\Delta t = 0.02 \cdot e^{0.74 \cdot mag} + 0.3 \cdot dist$$

[e.g., TRIFUNAC, 1975]

Δt from S- or P-pick for phase selection

Δt before origin-time for noise selection

[assuming noise is stationary]

Frequency domain: FFT

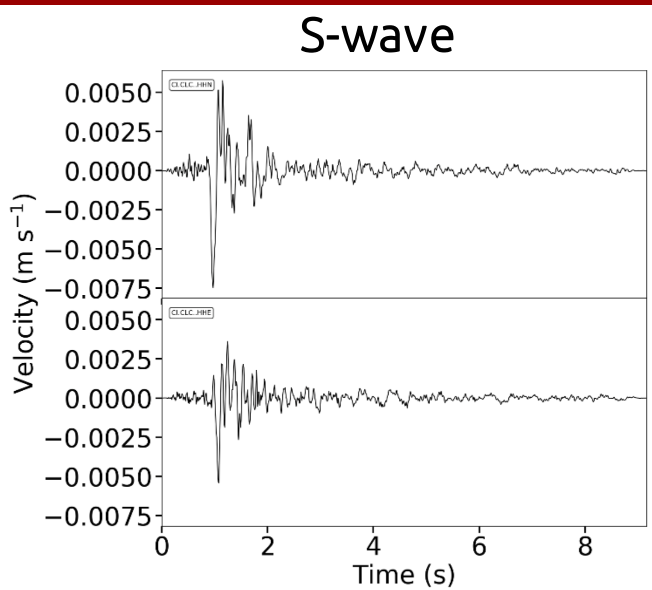
Select amplitude of the Fourier transform output

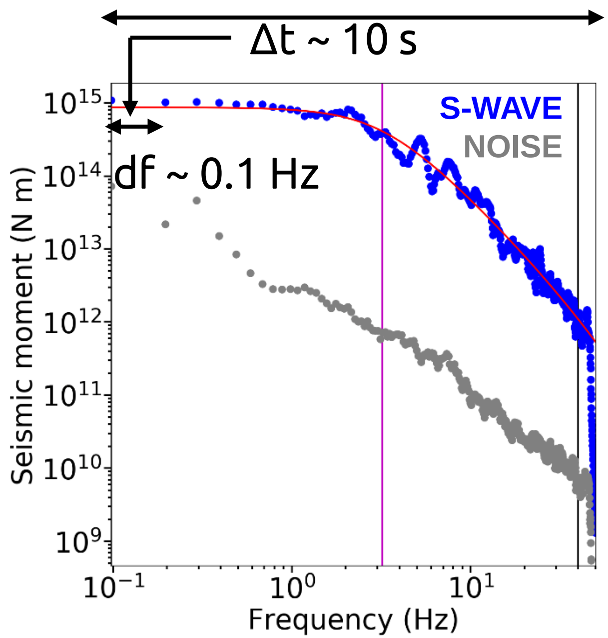
Integrate 1 or 2 times (if signal is velocity or acceleration) [integration is faster in frequency domain]

Displacement amplitude spectrum: observation to invert

From raw signals to source parameters

Pre-processing details





BEFORE FFT

- Remove constant and linear trend from signal and apply tapering function to first and last part (e.g. 5 %) of the data

Helps avoid introducing artificial frequencies in the FFT output

Δt must be large enough to have minimum FFT frequency df = 1/Δt smaller than corner frequency
(with some points on the left of fc …)

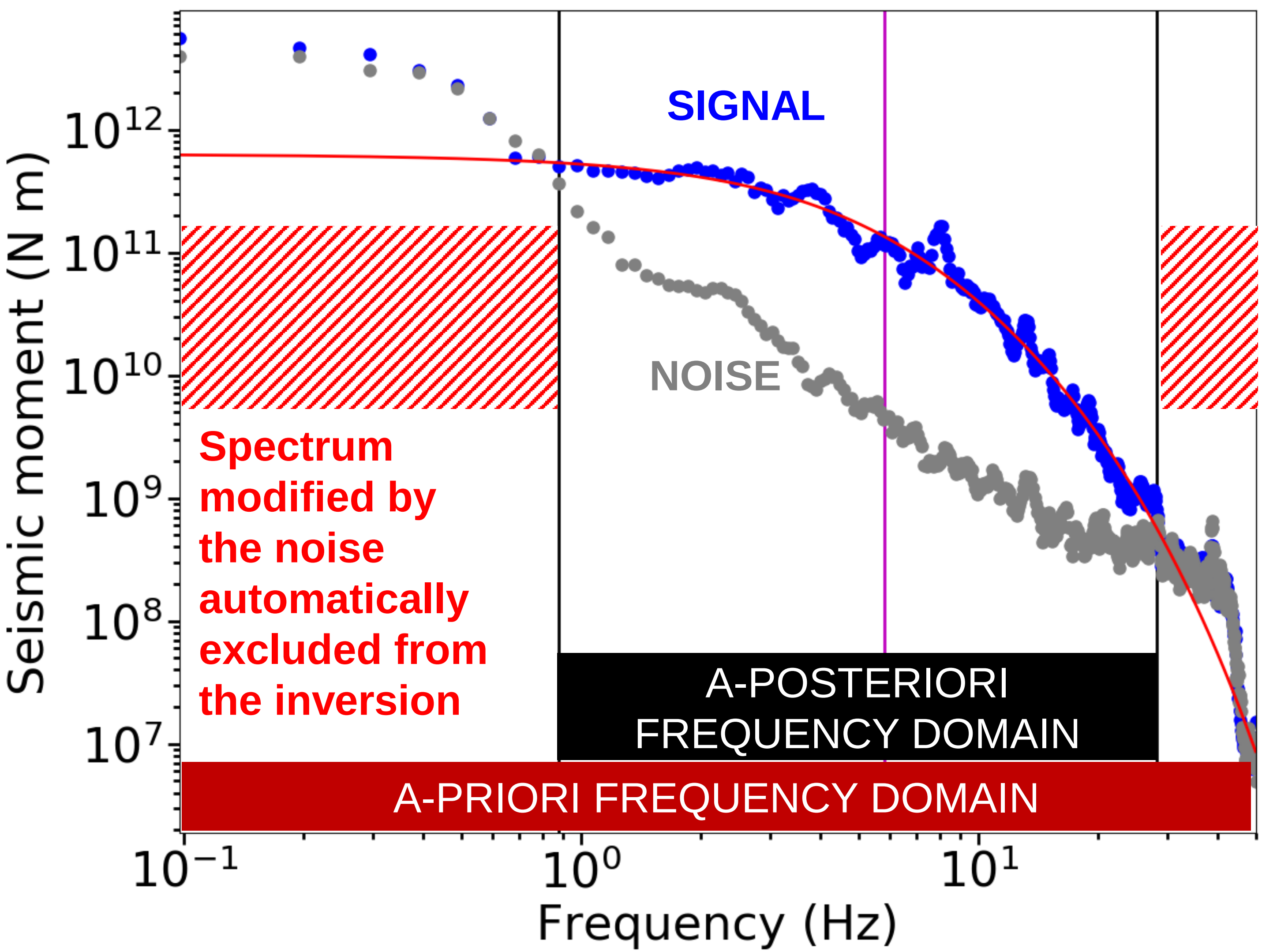
AFTER FFT

- Smooth the spectrum, e.g. 5-point moving average filter

Helps reduce the spikes produced by the FFT operation

From raw signals to source parameters
Frequency domain for the inversion

Signal-to-noise ratio (SNR) > threshold
to select inverted frequency domain



- DYNAMIC :

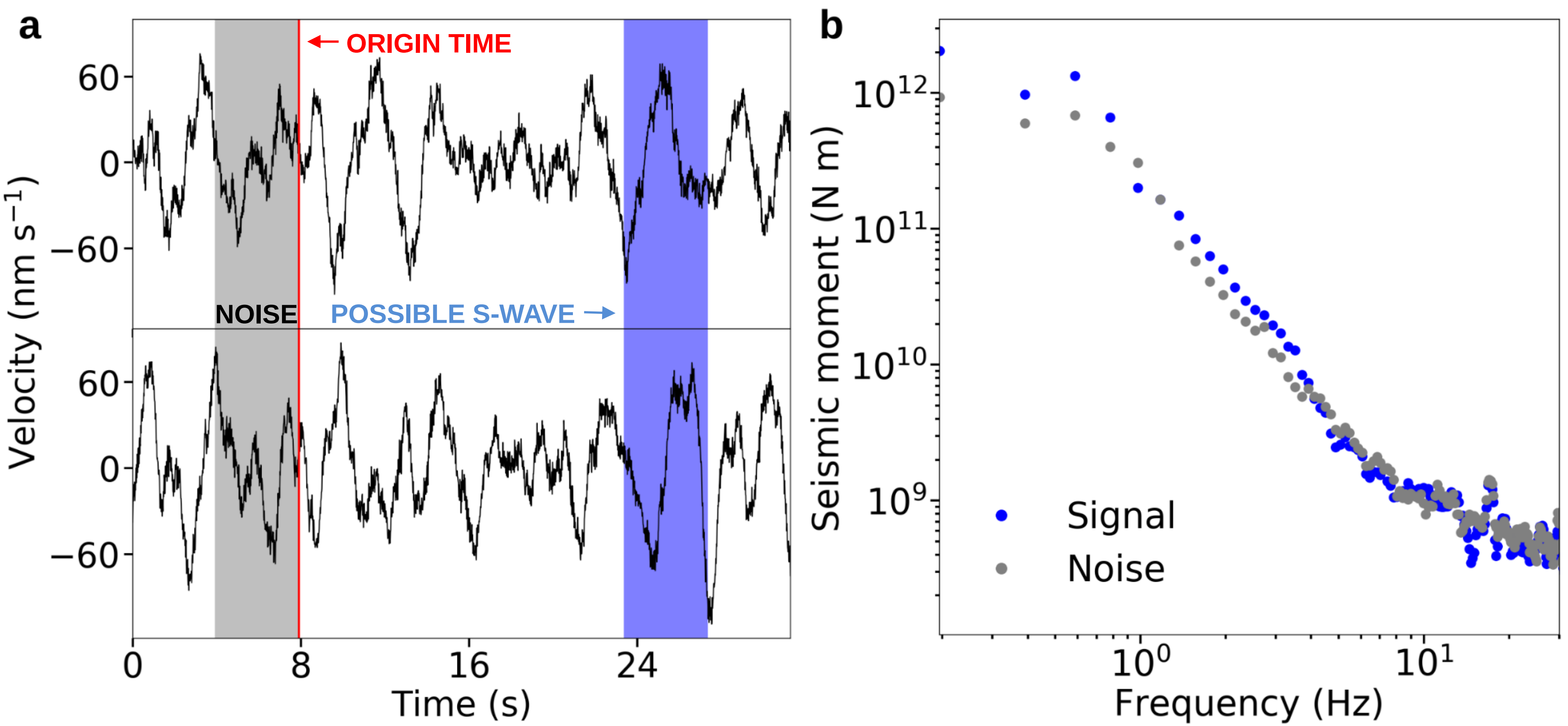
Accounts for SNR changes station by station, event by event

- ROBUST :

Returns an empty (or very small) set of frequencies if the S- or P-wave is wrongly selected, and it is noise

[Not rare in case of large analysis of automatic catalogs, especially for small events]

From raw signals to source parameters
Frequency domain for the inversion

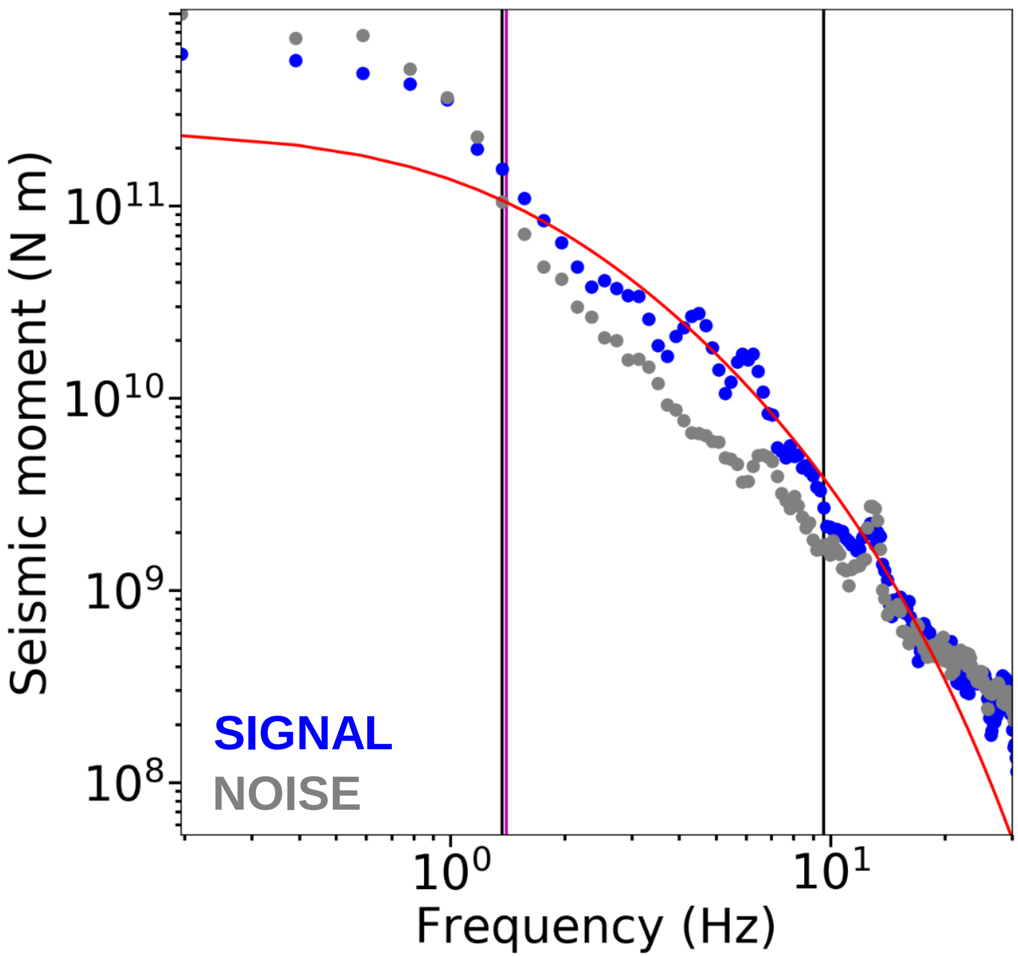


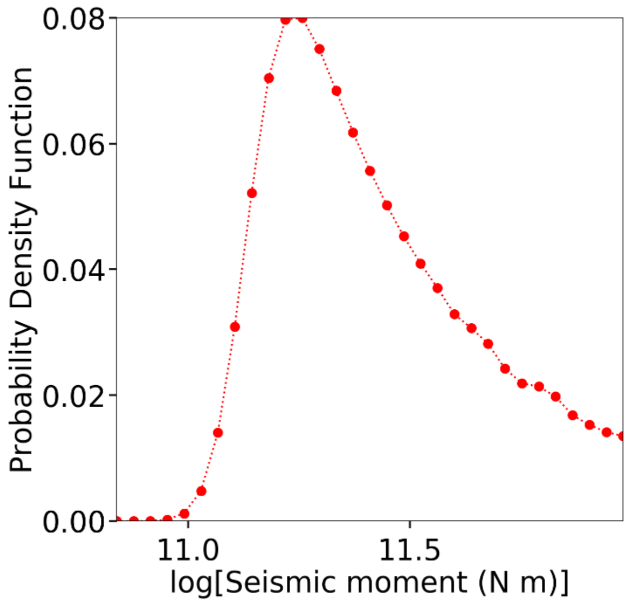
The a-posteriori frequency domain selected for the inversion is an empty set

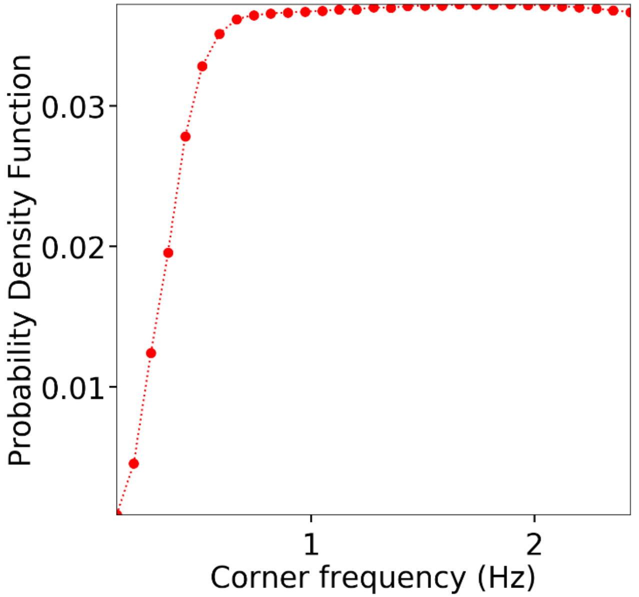
QUALITY OF THE SOLUTION

Quality of the solution

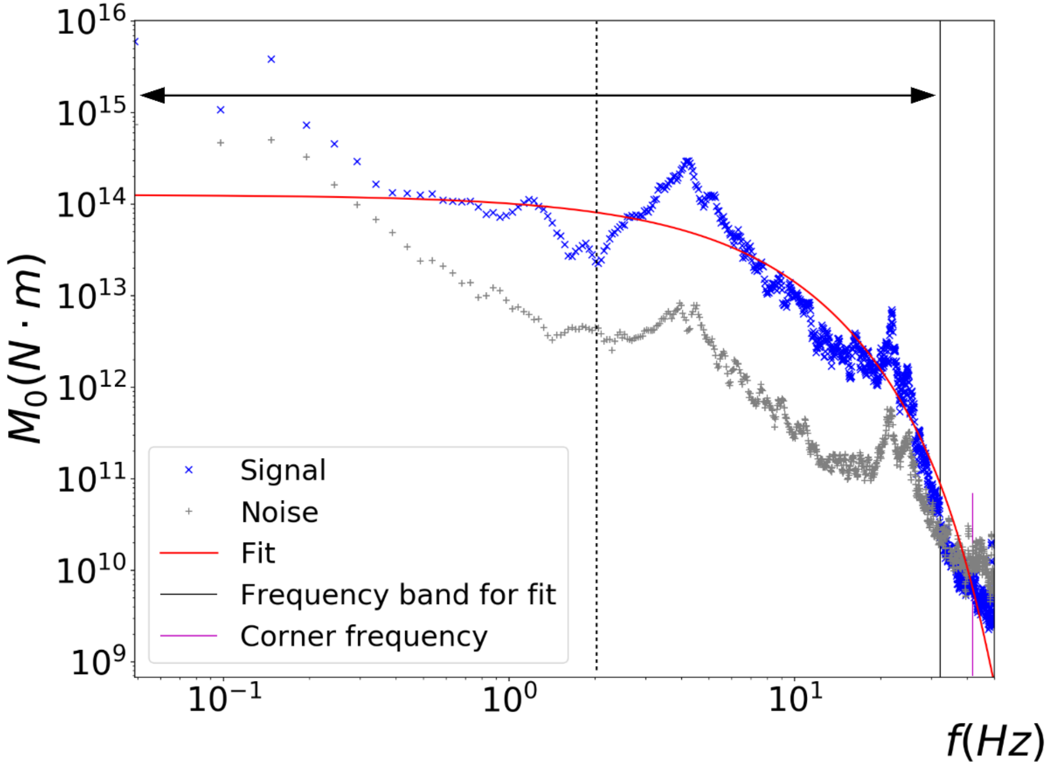
Unconstrained solutions | Low signal-to-noise ratio

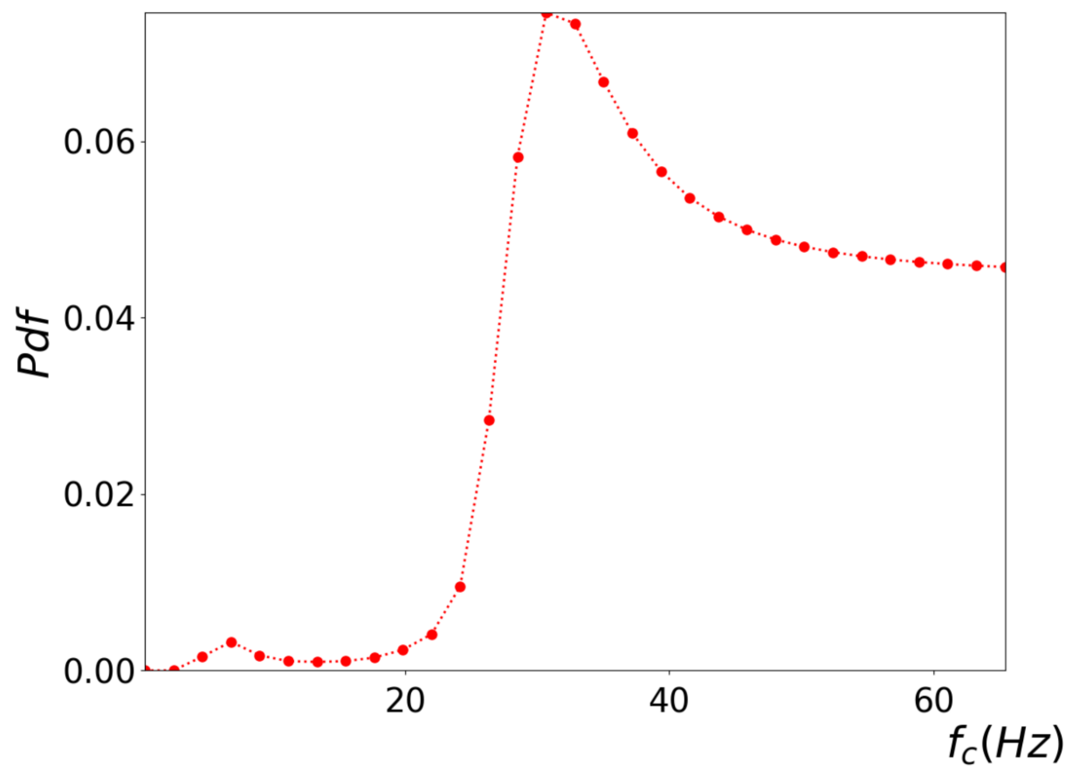






Quality of the solution
Unconstrained solutions | Spectral shape





Strong site amplification ~ 5 Hz

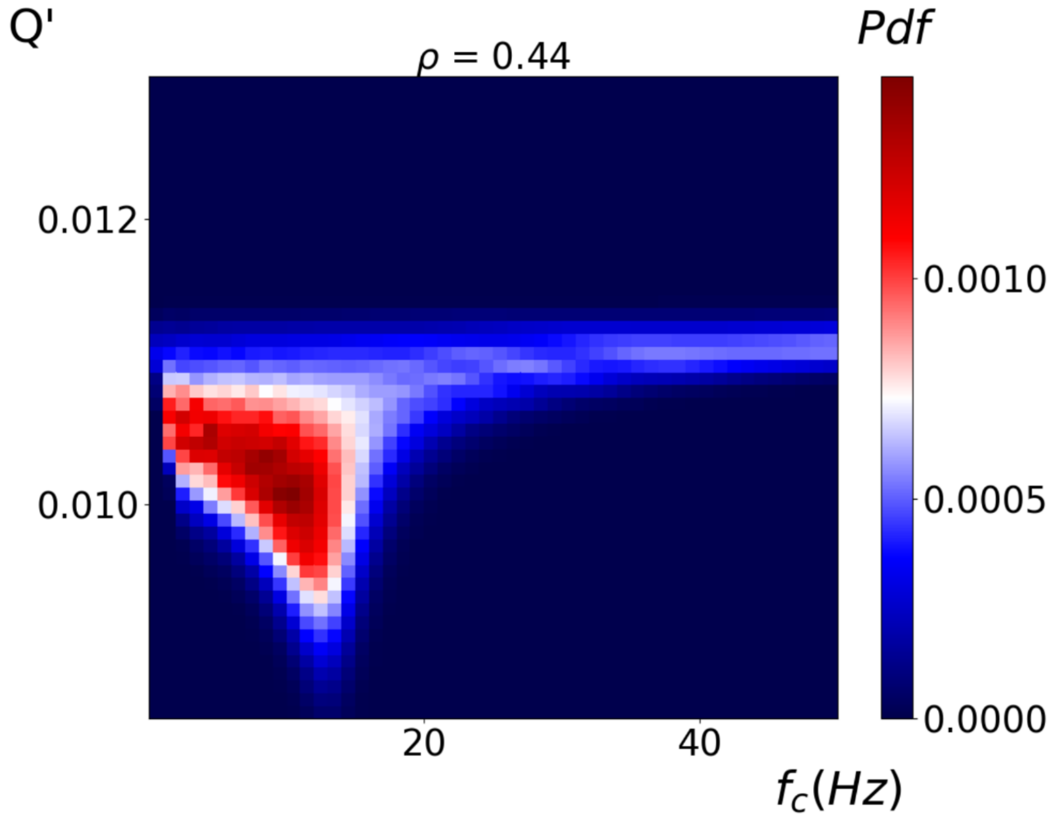
Spectral shape different from Brune model

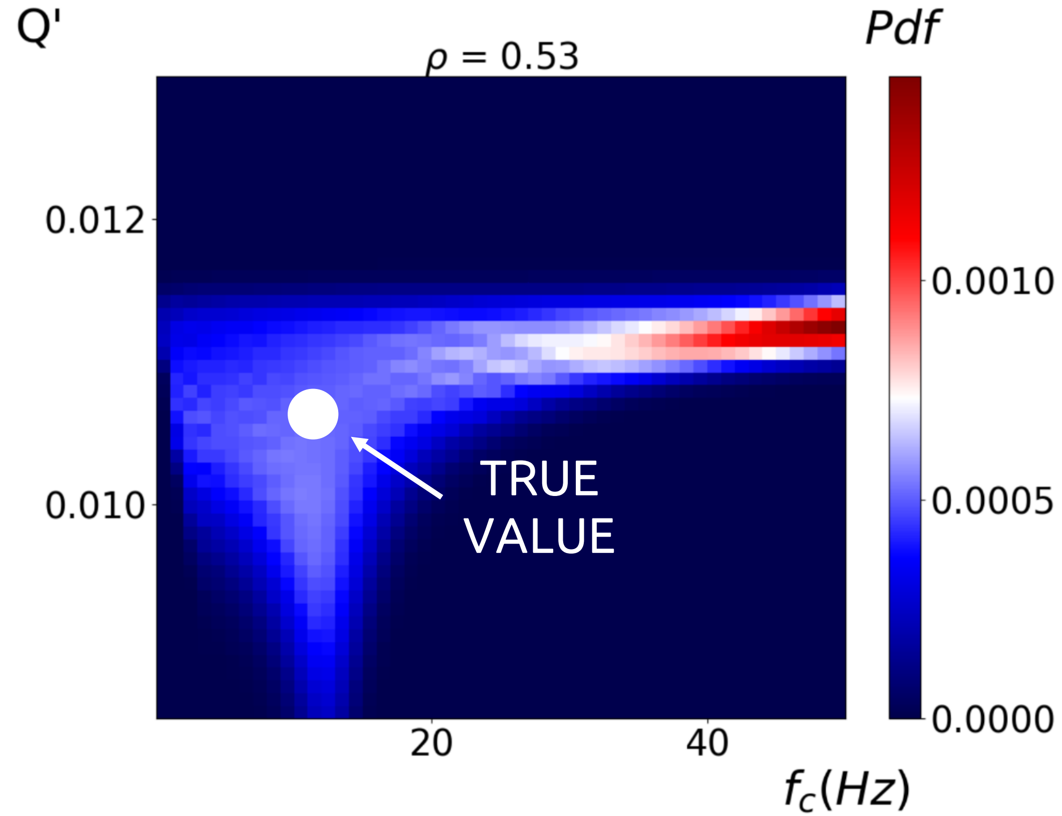
Unconstrained PDF solution for corner frequency

Quality of the solution

Unconstrained solutions | Frequency bandwidth sensitivity

Synthetic spectrum with fc = 10 Hz





0.5 Decades on the right of f_C

ACCEPTED SOLUTION

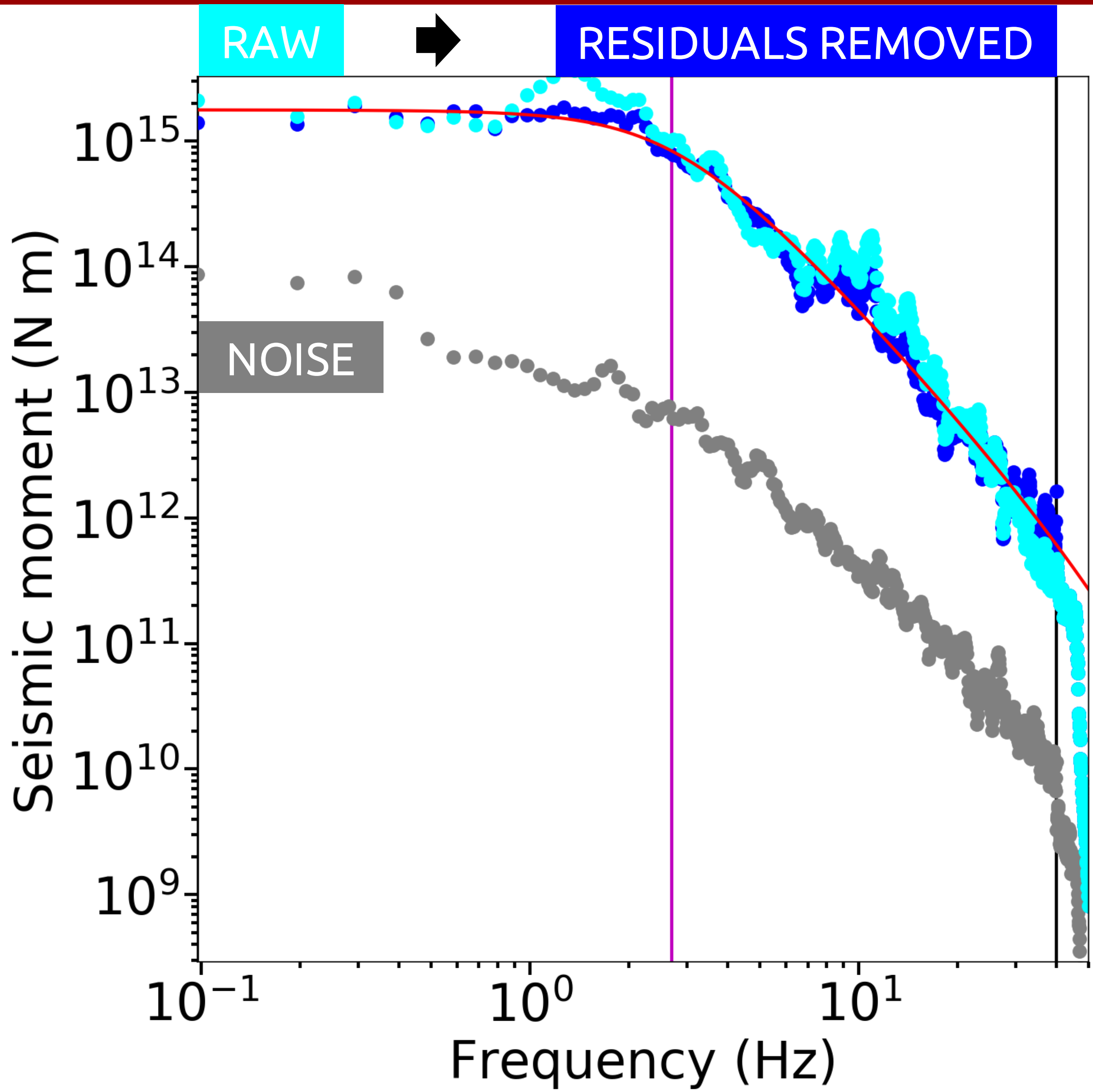
0.3 Decades on the right of f_C

REJECTED SOLUTION

SITE EFFECTS
SPECTRAL RESIDUALS REMOVAL
2-step procedure

Spectral residual removal

Single station



Residuals between single station observed spectrum and event best-fit spectrum

↓

Averaged among all events

↓

Removed from each single station observed spectrum

↓

Spectral inversion repeated on the corrected spectrum

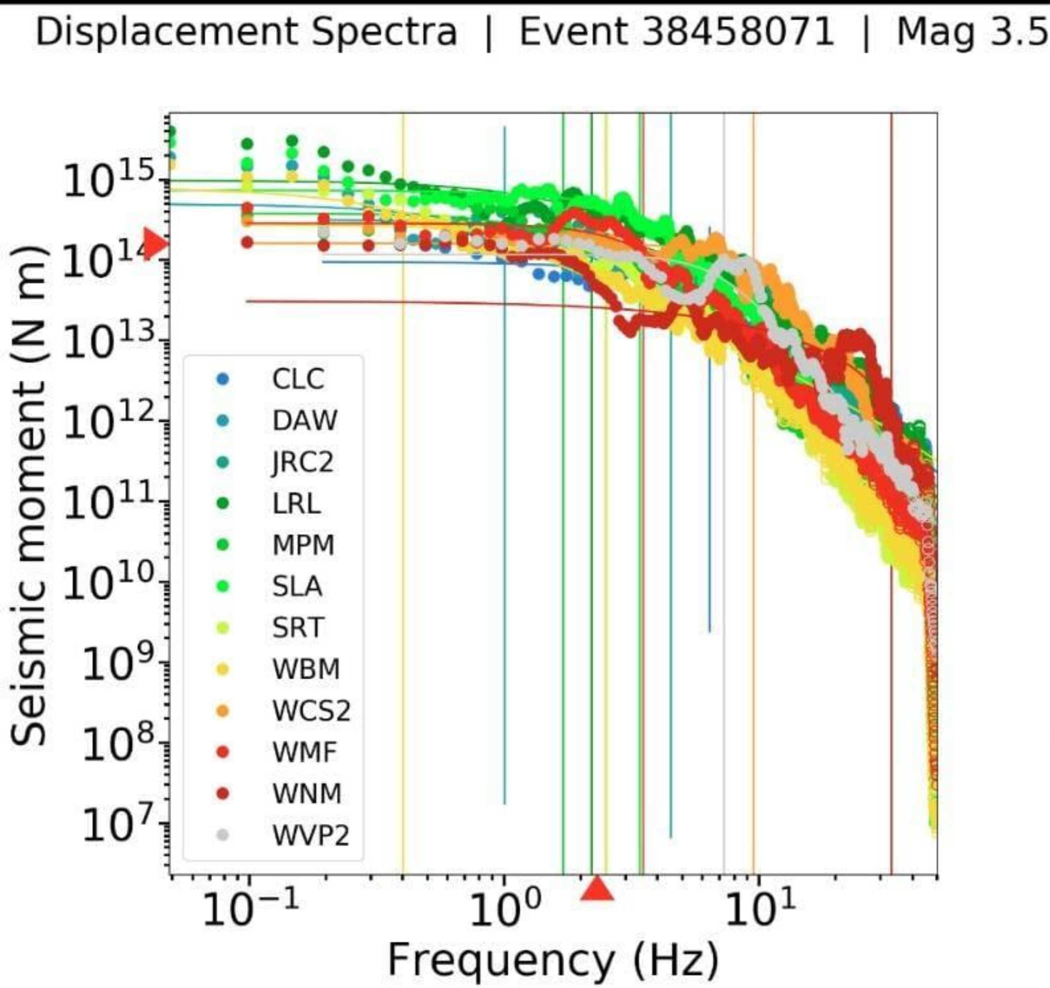
TARGET EVENT with a-priori information from residuals

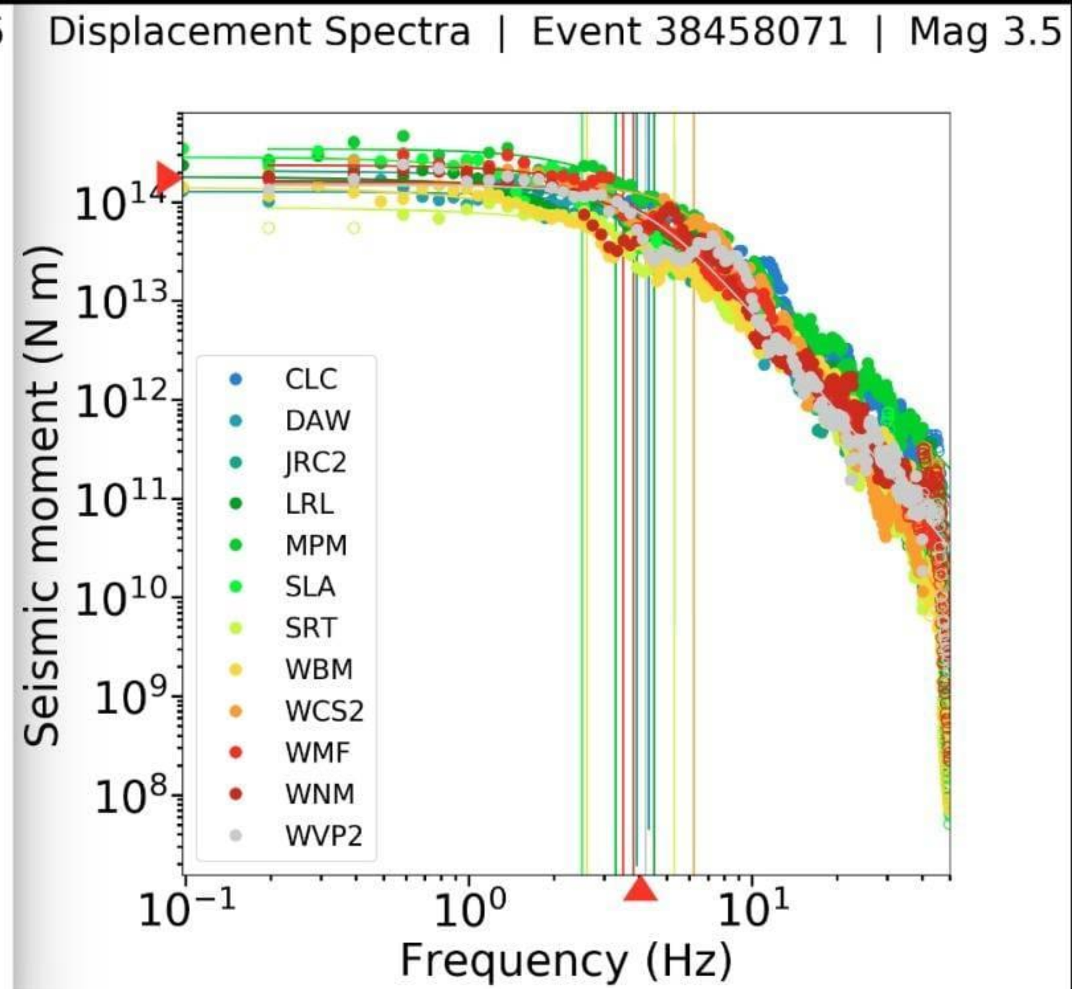
Suitable for near real-time inversions (seismic monitoring)

Spectral residual removal

Single event, all stations

RAW → RESIDUALS REMOVED



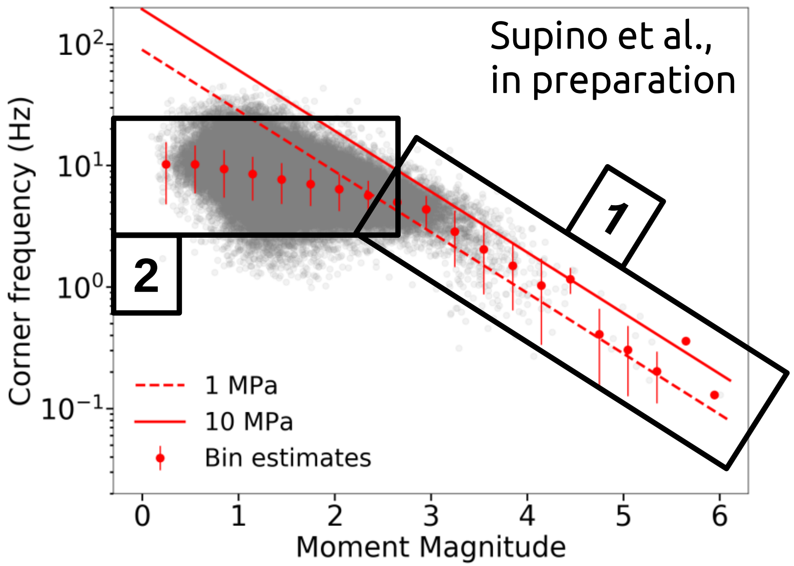


Average estimates do not change much, especially Mw

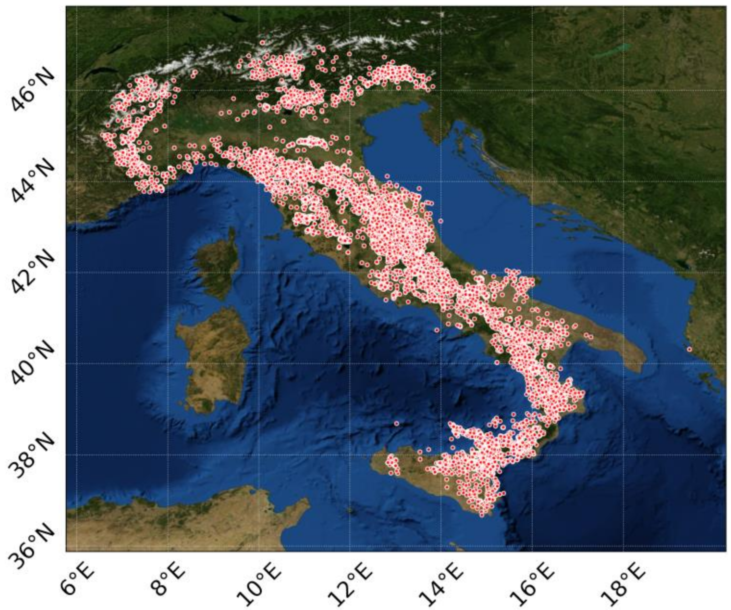
Variability (uncertainty) is strongly reduced

SINGLE-STATION APPROACH
LIMITATIONS AND BENEFITS

Single station approach
Limitations and Benefits



1	Almost constant stress drop (1-10 Mpa) Mw >~ 2.5
2	Almost constant $f_c \approx$ 10 Hz Mw <~ 2.5



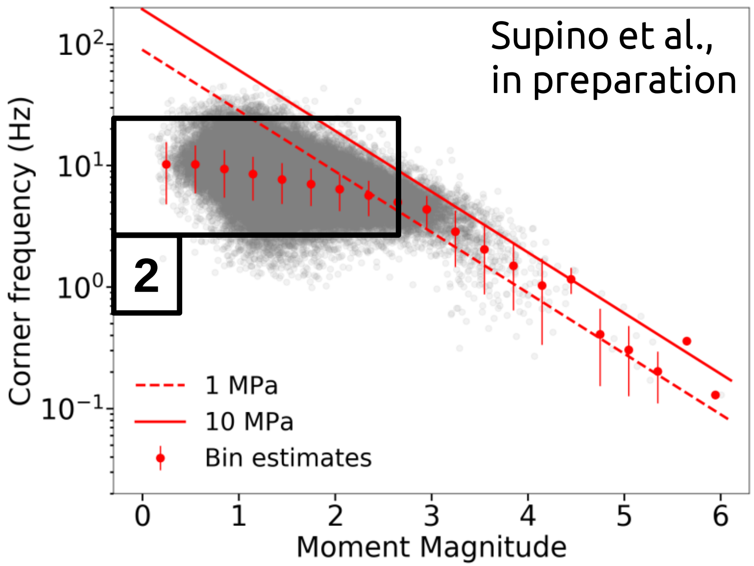
INGV Catalog

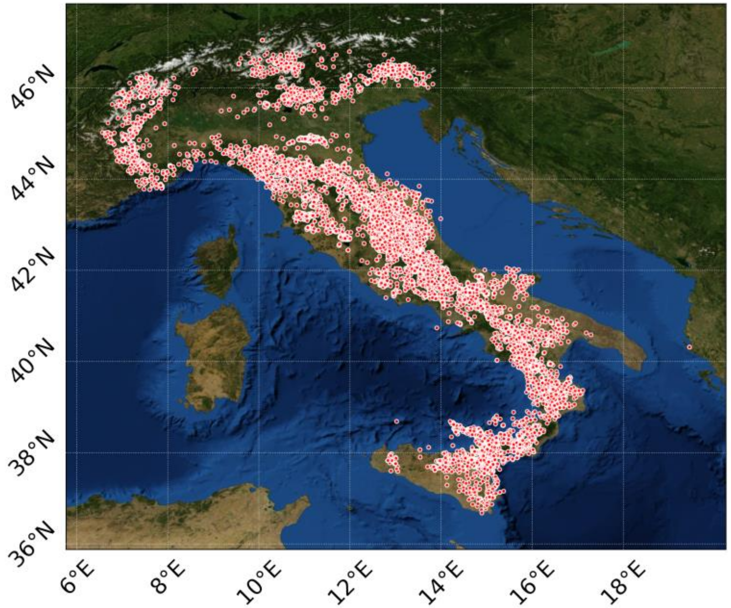
10 years | 2009 - 2018

~250,000 events

~2,400,000 S-waves inverted

Single station approach
Limitations and Benefits





2

Mw <~ 2.5

Almost constant $f_c \approx 10$ Hz

Apparent f_c due to anelastic attenuation low-pass filtering

f_c is the cut-off frequency of the low-pass filter and does not scale with the source

e.g. Abercrombie, 1995
Ide et al., 2003
...

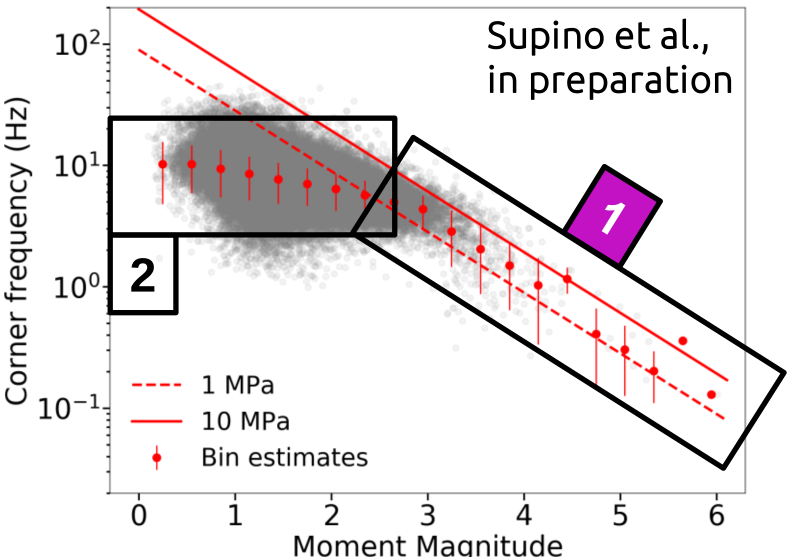
INGV Catalog

10 years | 2009 - 2018

~250,000 events

~2,400,000 S-waves inverted

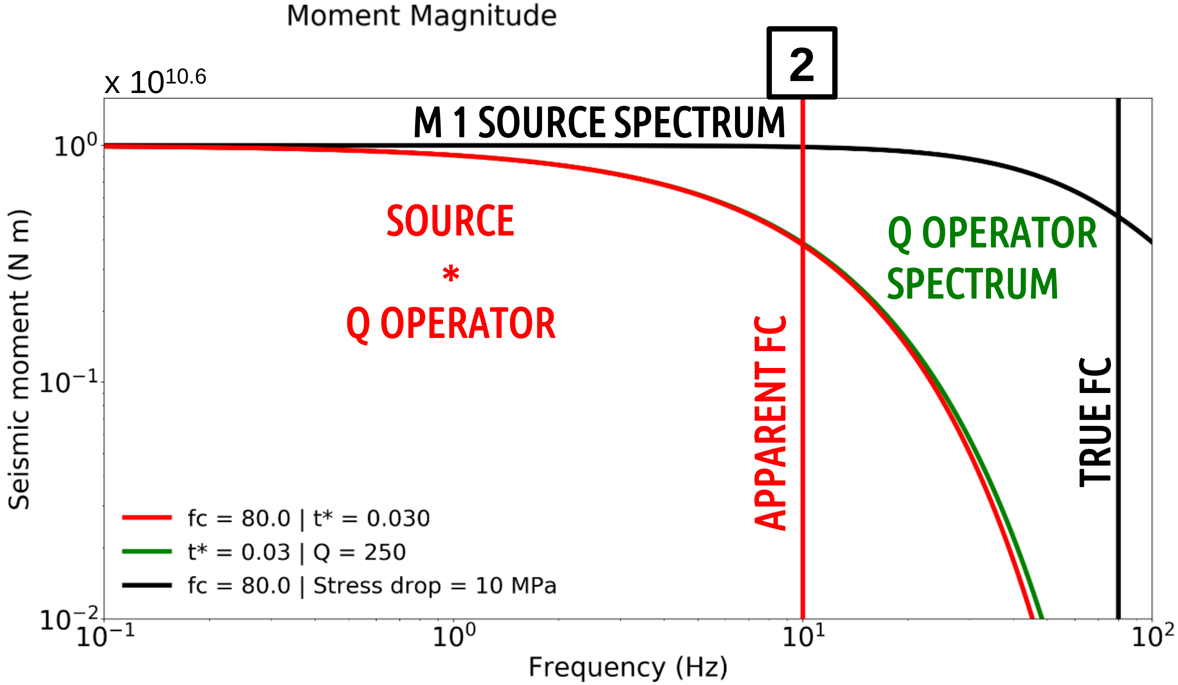
Single station approach
Limitations and Benefits

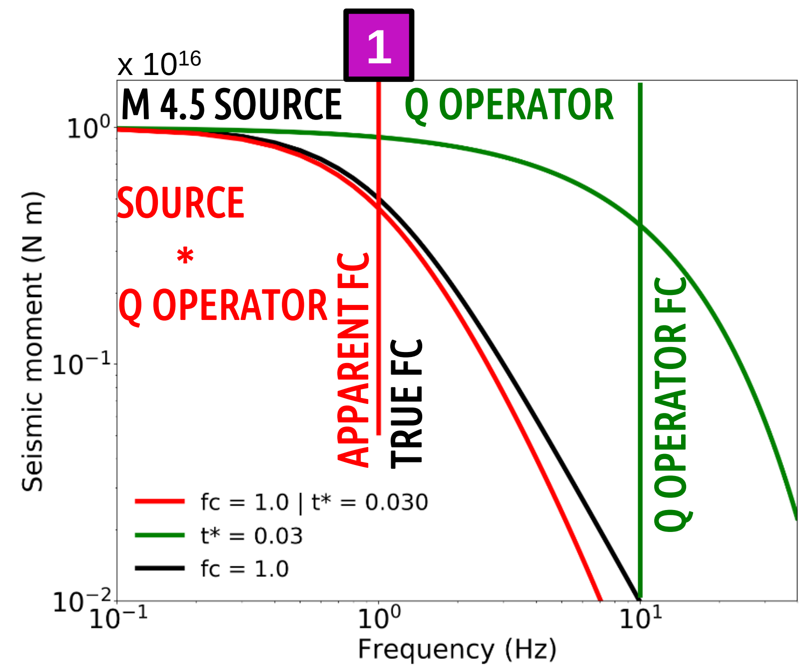


M <~ 2.5: Estimation of Mw **2**

M ~ 2.5 – 6: Estimation of Mw, fc, stress drop **1**

Near-real time application
for seismic monitoring purposes





STRESS DROP APPLICATION

Campi Flegrei Mw 3.7 earthquake
20 May 2024

REPORT
doi:10.26443/seismica.v3i2.1394
SEISMICA

Source characterization of the 20th May 2024 M_D 4.4 Campi Flegrei caldera earthquake through a joint source-propagation probabilistic inversion

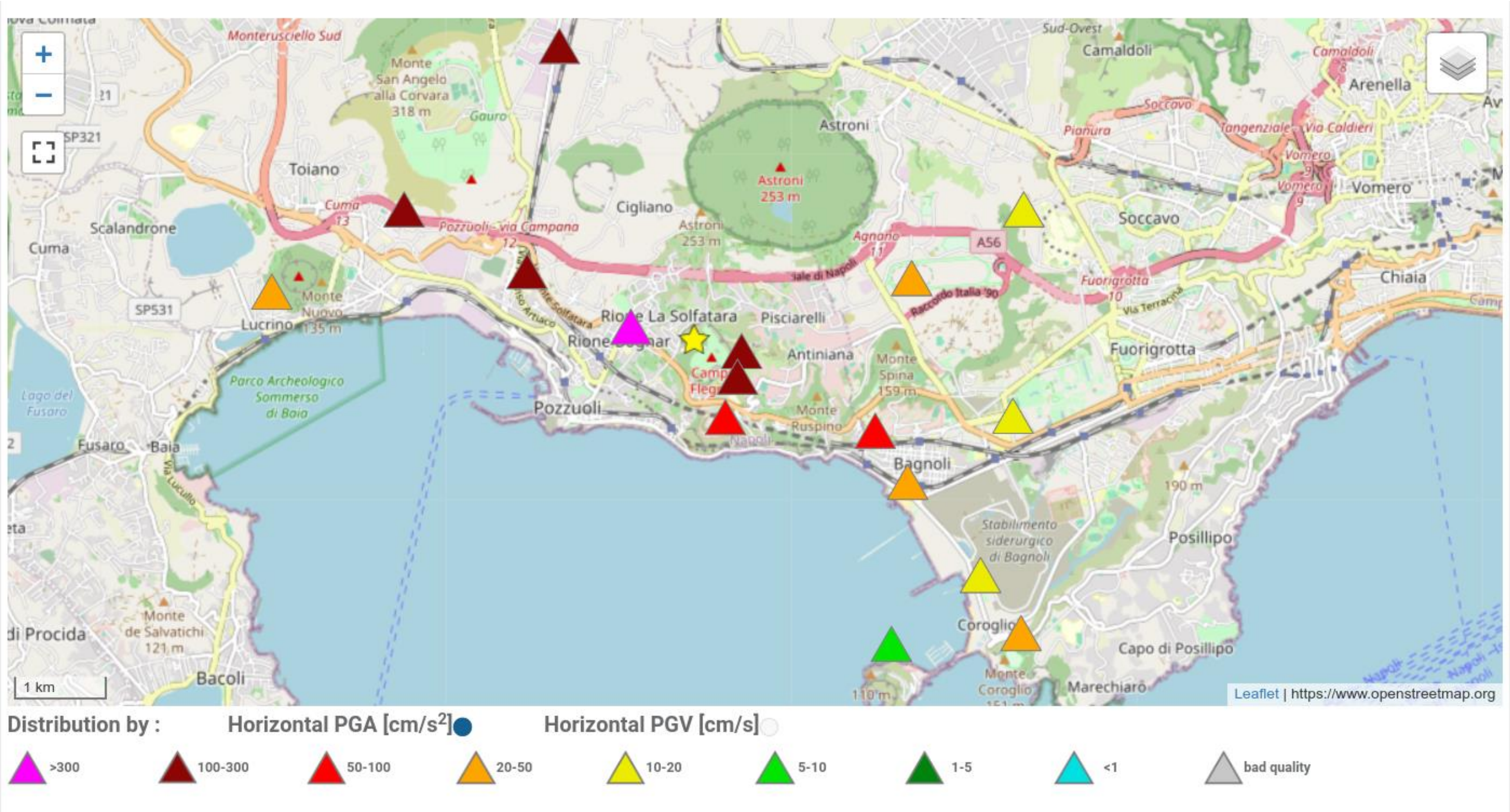
M. Supino *[1], **L. Scognamiglio** [1], **L. Chiaraluce** [1], **C. Doglioni** [1], **A. Herrero** [1]

[1] Istituto Nazionale di Geofisica e Vulcanologia, Roma, Italy

Campi Flegrei Mw 3.7 May 20 2024 earthquake

Observed PGA

POZZUOLI Mw 3.7 | PGA MAX 358 cm s^{-2}



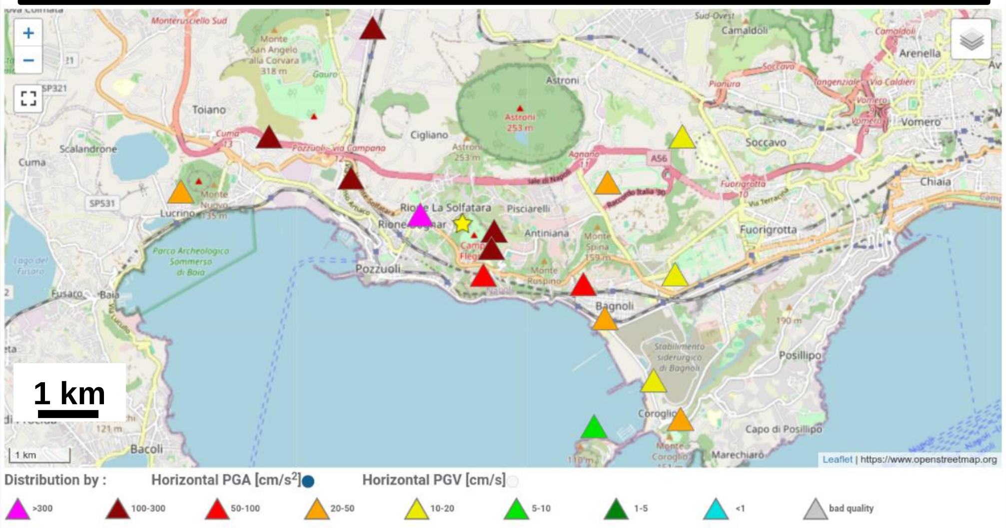
Largest earthquake ever recorded in Campi Flegrei caldera

Shallow depth 2.6 km

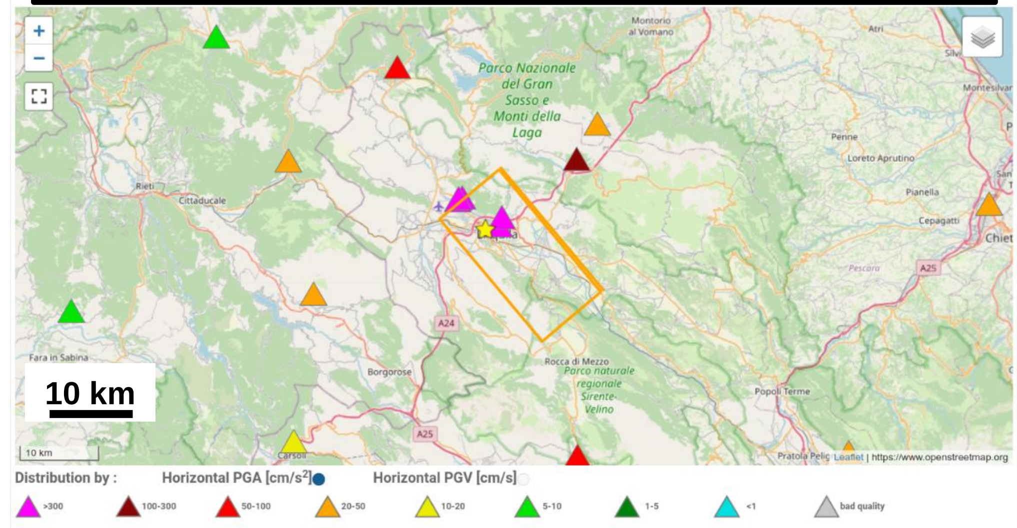
Campi Flegrei Mw 3.7 May 20 2024 earthquake

Observed PGA

POZZUOLI Mw 3.7 | PGA MAX 358 cm s^{-2}



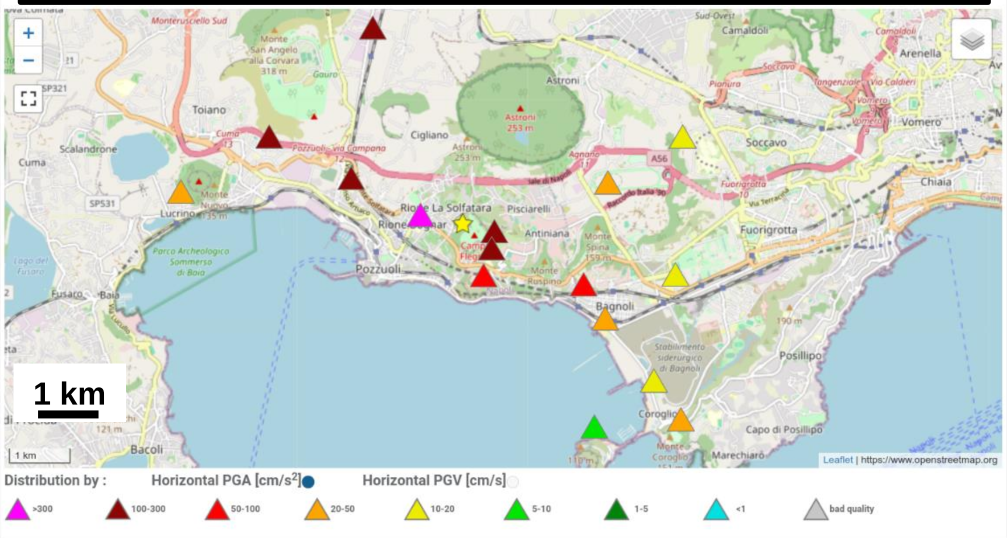
L'AQUILA Mw 6.1 | PGA MAX 644 cm s^{-2}



Campi Flegrei Mw 3.7 May 20 2024 earthquake

Observed PGA

POZZUOLI Mw 3.7 | PGA MAX 358 cm s^{-2}



Can we infer some constrains on the possible increase in magnitude ?

EXPECTED MAX PGA ?

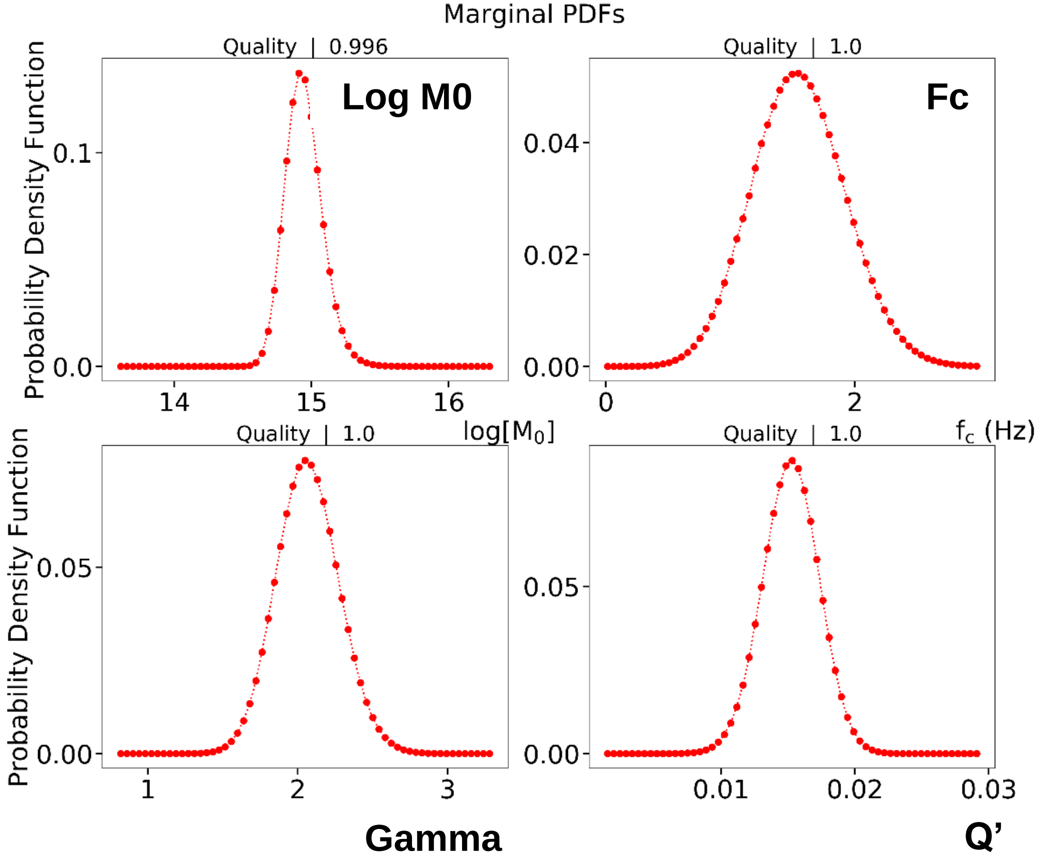
EXPECTED BUILDINGS RESPONSE ?

~200,000 people in the represented area

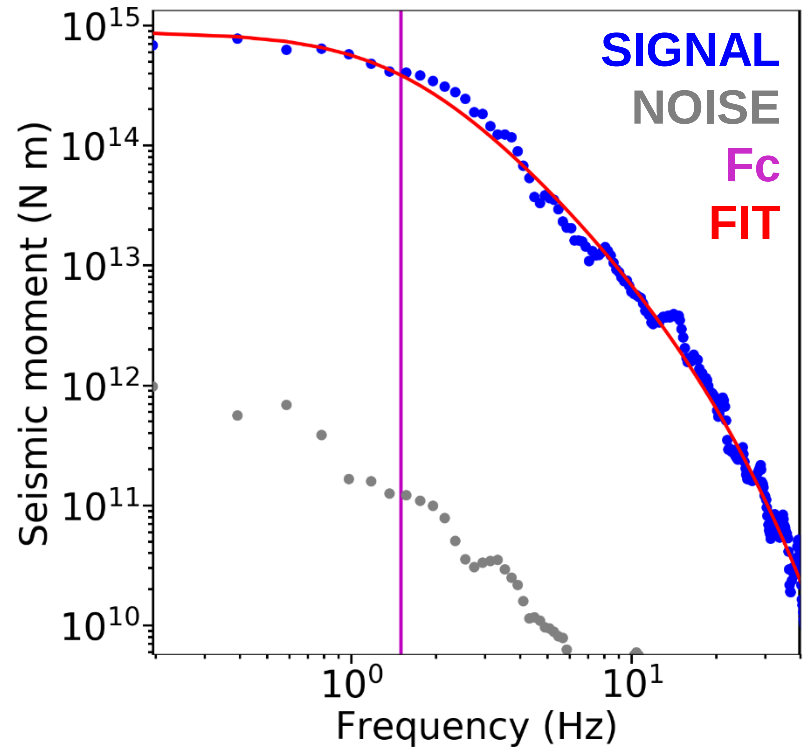
Campi Flegrei Mw 3.7 May 20 2024 earthquake

Supino et al., 2024 – Seismica | Link

1d Marginals from integration of 4d Joint PDF



Displacement spectrum in M0 units



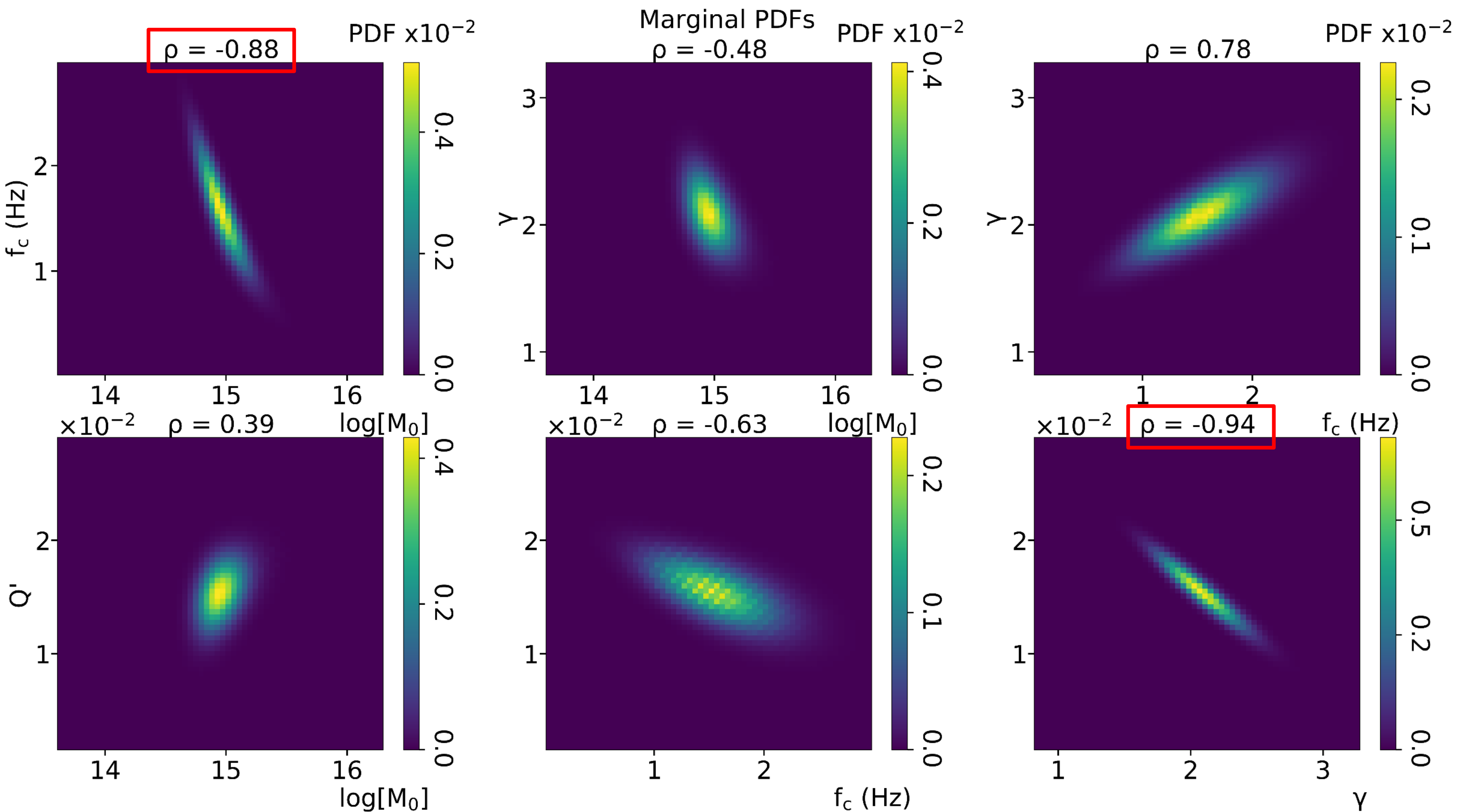
Log M0	Mw	Fc (Hz)	Gamma	Q	t*
14.95 ± 0.13	3.89 ± 0.09	1.6 ± 0.4	2.1 ± 0.2	66 ± 9	0.0407

1-sigma confidence level

Campi Flegrei Mw 3.7 May 20 2024 earthquake

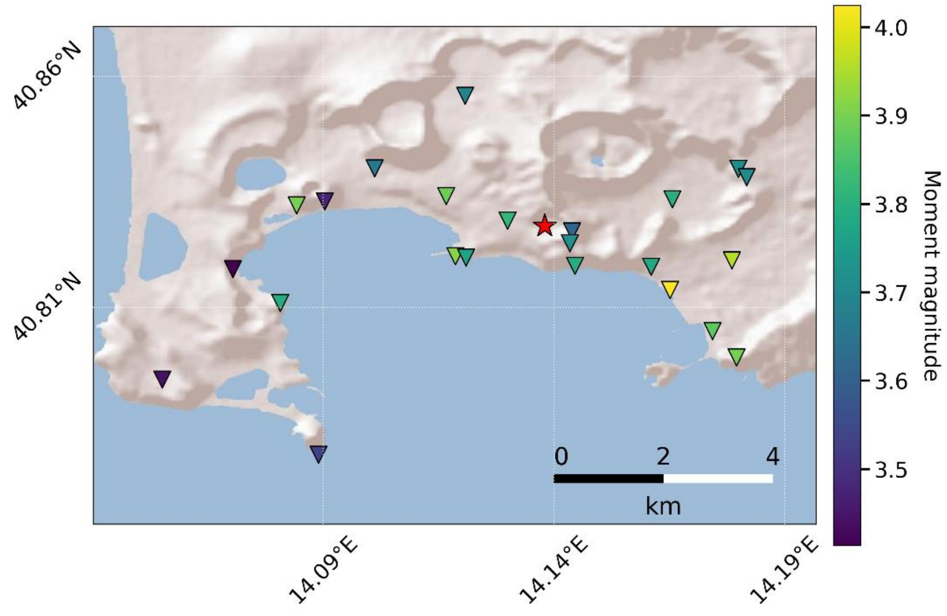
Supino et al., 2024 – Seismica | Link

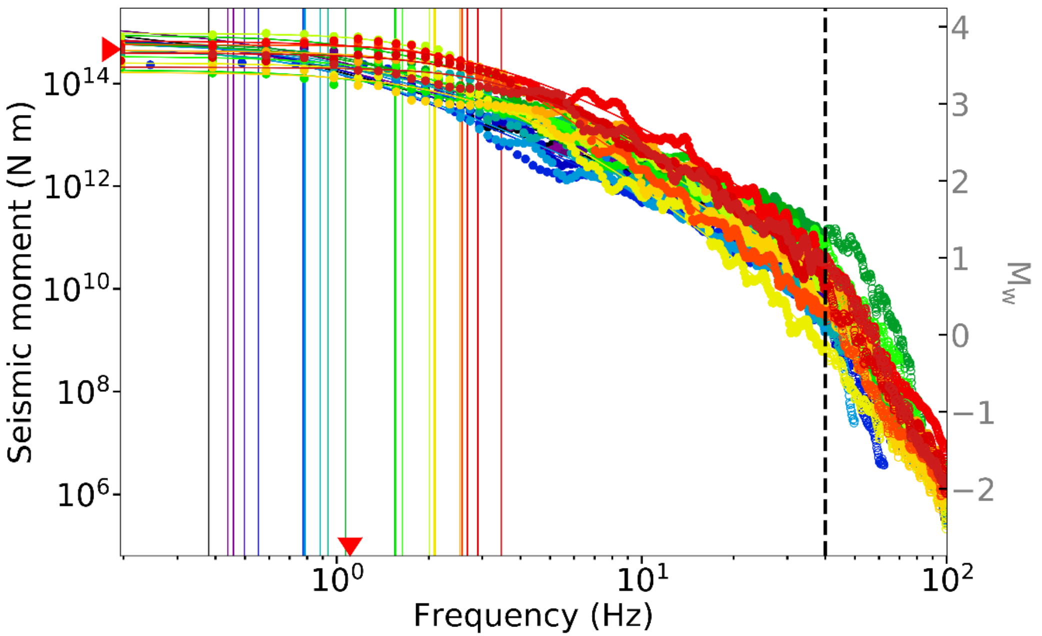
2d Marginals from integration of 4d Joint PDF



Campi Flegrei Mw 3.7 May 20 2024 earthquake

Supino et al., 2024 – Seismica | Link





23 Stations | Hypocentral distance = 2.8 – 8.3 km

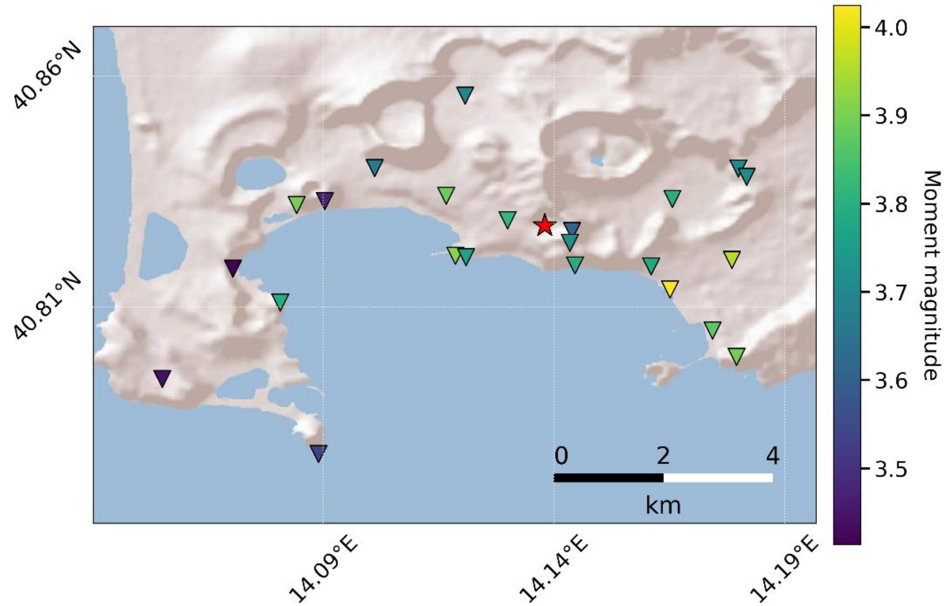
Mw	Fc (Hz)	Gamma	Q
3.70 ± 0.13	**1.11 ± 0.19**	**2.07 ± 0.1**	**70 ± 20**

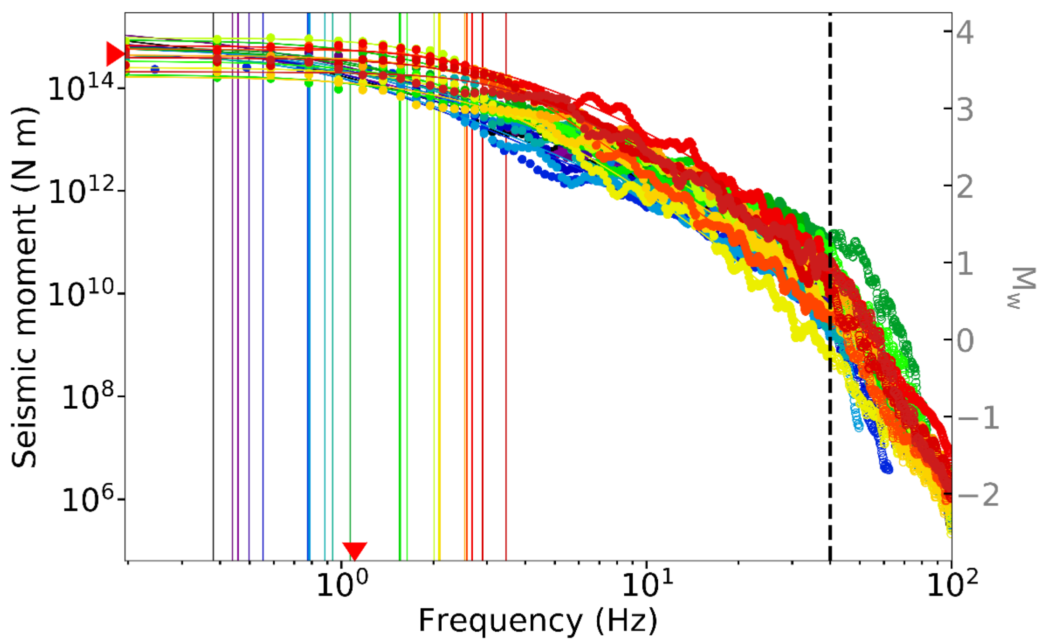
ASSUMING
V_R = 0.9 V_S and Kaneko and Shearer (2014) circular source model | **k = 0.26**

$$\Delta\sigma = 3.2 \pm 2.2\ MPa$$
$$r = 400 \pm 70\ m$$

Campi Flegrei Mw 3.7 May 20 2024 earthquake

Supino et al., 2024 – Seismica | Link





From the estimated $\Delta\sigma$
assuming a maximum rupture length from seismicity distribution
(r_{MAX} = 1.5 km, e.g. Danesi et al., 2024)

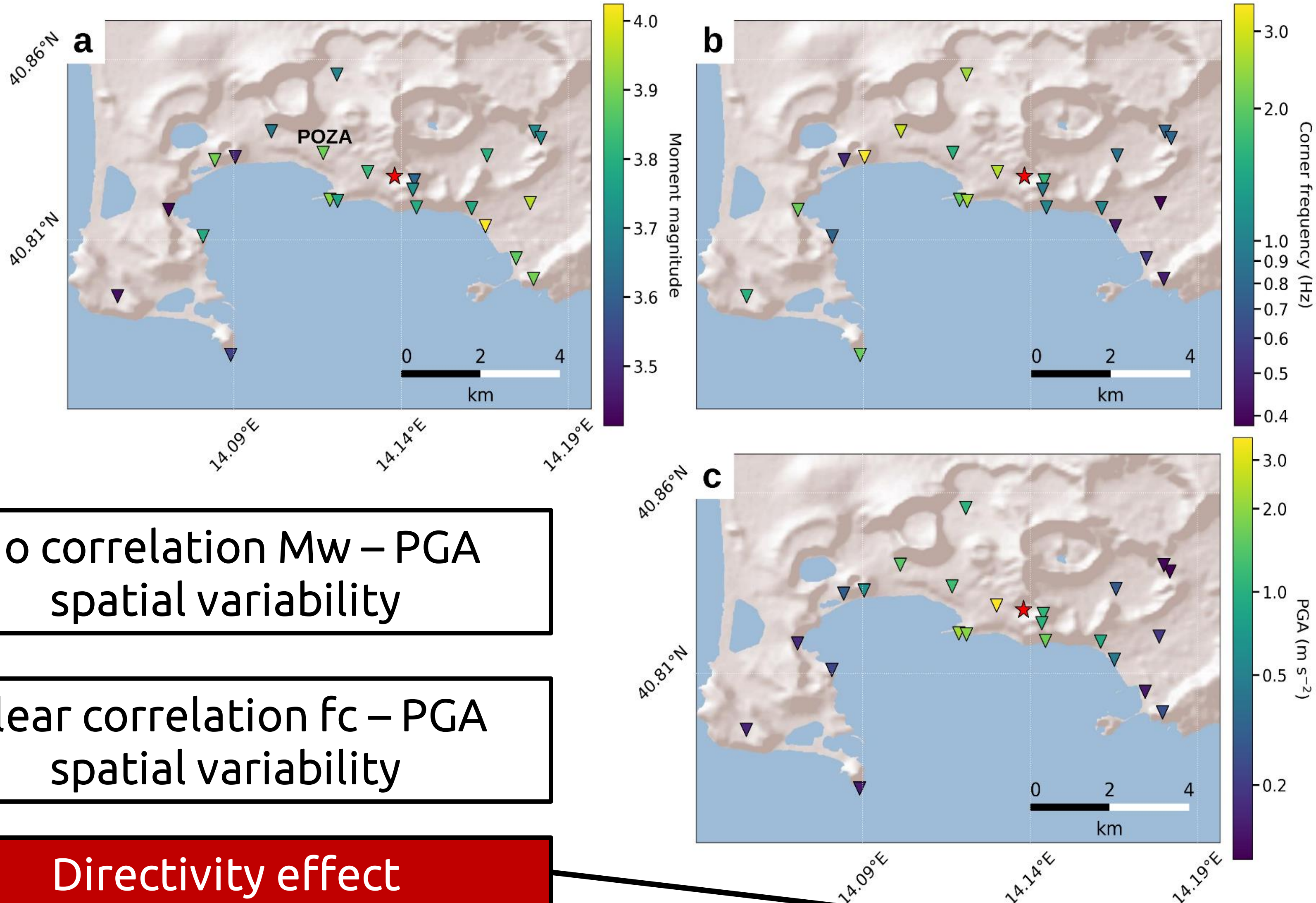
$$M_0^{MAX} = \frac{16}{7}\Delta\sigma \cdot r_{MAX}^3$$

$\boldsymbol{M_w^{MAX} = 4.9 \pm 0.3}$ $\boldsymbol{\Delta\sigma = 3.2 \pm 2.2\ MPa}$

Possible scenarios (PGA) from GMPEs | INPUT: $\boldsymbol{M_w}$, $\boldsymbol{\Delta\sigma}$

Campi Flegrei Mw 3.7 May 20 2024 earthquake

Supino et al., 2024 – Seismica | Link



No correlation Mw – PGA spatial variability

Clear correlation fc – PGA spatial variability

Directivity effect

Possible scenarios (PGA) from GMPEs | INPUT: M_w, $\Delta\sigma$ +

SOFTWARE

SourceSpec software

Claudio Satriano | IPGP | Link

SourceSpec

latest

Search docs

- Theoretical Background
- Signal Processing
- Clipping Detection
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SourceSpec documentation

Earthquake source parameters from P- or S-wave displacement spectra

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SourceSpec is a collection of command line tools to compute earthquake source parameters (seismic moment, corner frequency, radiated energy, source size, static stress drop, apparent stress) from the inversion of P-wave and S-wave displacement spectra recorded at one or more seismic stations. SourceSpec also computes attenuation parameters (t-star, quality factor) and, as a bonus, local magnitude.

See Madariaga [2011] for a primer on earthquake source parameters and scaling laws.

Go to section Theoretical Background to get more information on how the code works.

Method similar to SPAR | GitHub | Python

Extensive documentation on both code and theoretical background

SourceSpec software

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SourceSpec

SourceSpec

latest

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Using pip and PyPI

The latest release of SourceSpec is available on the Python Package Index.

You can install it easily through `pip`:

```
pip install sourcespec
```

To upgrade from a previously installed version:

```
pip install --upgrade sourcespec
```

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The war which is coming
Is not the first one.

There were other wars before it.
When the last one came to an end
There were conquerors and conquered.

Among the conquered the common people
Starved. Among the conquerors
The common people starved too.

Bertolt Brecht

Thank you