Putting 3D Dynamic Rupture Modeling in the Context of 3D Earthquake Cycle Simulations

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2024 SCEC Dynamic Rupture Workshop
November 4, 2024

Outline

- **Brief History:** Duan’s perspective on Dynamic rupture modeling

- **Dynamic Rupturing within Earthquake Cycles**: One Future Direction

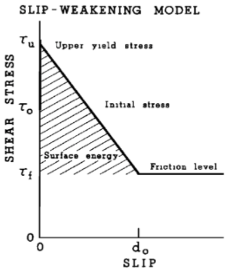
- **Concluding Remarks**

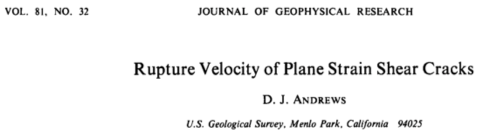
Dynamic Rupture Modeling

1. BRIEF HISTORY: DUAN’S PERSPECTIVE

Classical Paper #1 on Methodology

- Andrews (1976), Rupture Velocity of Plane Strain Shear Cracks, JGR.
- 2D spontaneous rupture.




VOL. 81, NO. 32 JOURNAL OF GEOPHYSICAL RESEARCH

Rupture Velocity of Plane Strain Shear Cracks

D. J. Andrews

U.S. Geological Survey, Menlo Park, California 94025

- Rupture propagation:
  - Supershear transition: Fig 3

Leading edge: separating zero from nozero slip velocity
Trailing edge: where slip equals d_0, stress drop is complete

- Snapshots

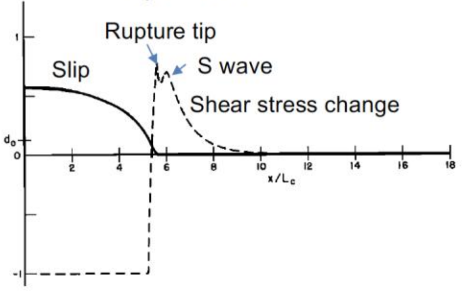


Fig. 4. Dynamic solution as a function of position on the crack plane at the dimensionless time $\beta t/L_c = 8.07$. Solid curve is dimensionless slip function divided by 10, $\mu\Delta u/[10L_c(\tau_0 - \tau_f)]$; dashed curve, dimensionless change of shear stress, $(\tau_{xy} - \tau_0)/(\tau_0 - \tau_f)$.

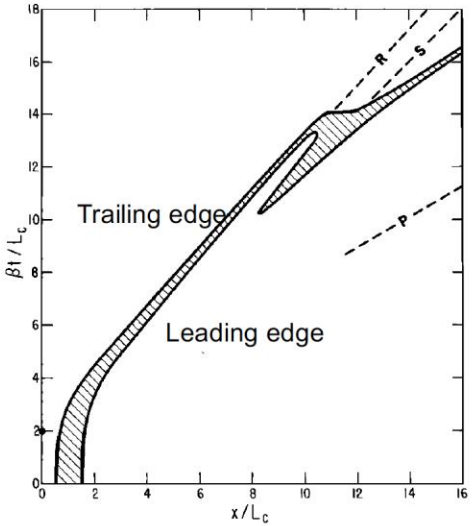


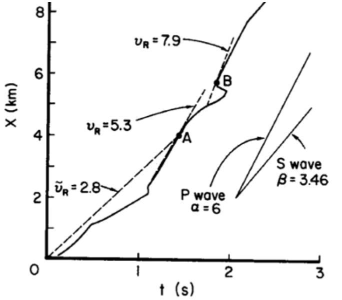
Fig. 3. Space-time plot of rupture propagation in dynamic calculation starting from nearly static solution of Figure 2. Region between the two solid lines is the rupture front, where slip velocity is nonzero and stress drop is incomplete. Dashed lines labeled R, S, and P, drawn for reference with slopes corresponding to Rayleigh, shear, and compressional wave velocities, respectively, diverge from the solid point on the time axis.

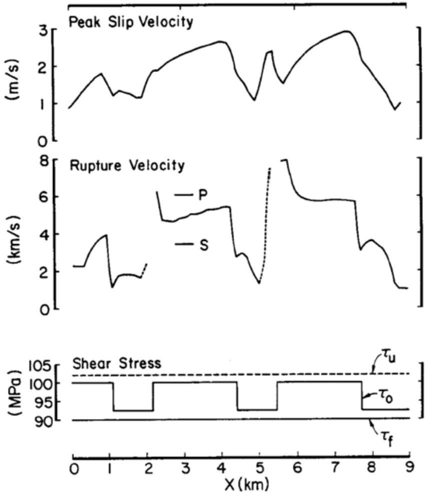
Classical Paper #2 on Methodology

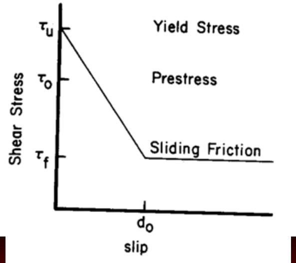
- Day (1982), Three-dimensional simulation of spontaneous rupture: The effect of nonuniform prestress, BSSA.

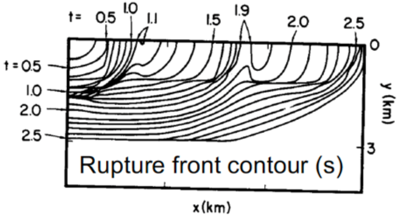
- 3D spontaneous rupture.

- Result #4: 5 high-stress patches
  - Complex rupture front: stop (1 s)/recommence (1.8 s) along y-axis; "jump" at 1.1 s, 1.9 s along x.
  - Close relation between peak slip V and local rupture V.
  - apparent faster than P local rupture V due to jump (out of sequence); "secant" rupture V locally supershear, but subshear over the entire fault length.









Rupture front contour (s)

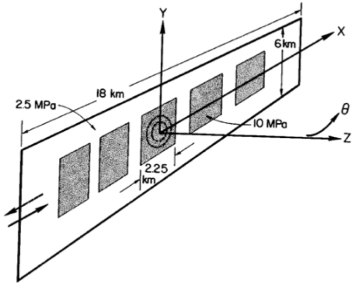
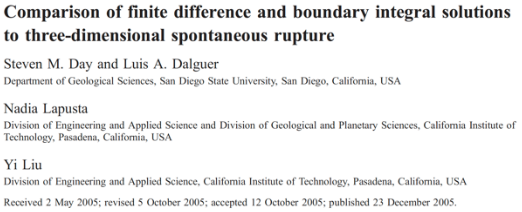


FIG. 12. Fault geometry for nonuniform prestress, problem III.

Classical Paper #3 on Methodology

- Day et al (2005), Comparison of finite different and boundary integral solutions to three-dimensional spontaneous rupture, JGR.
- TSN implementation.



Comparison of finite difference and boundary integral solutions to three-dimensional spontaneous rupture

Steven M. Day and Luis A. Dalguer
Department of Geological Sciences, San Diego State University, San Diego, California, USA

Nadia Lapusta
Division of Engineering and Applied Science and Division of Geological and Planetary Sciences, California Institute of Technology, Pasadena, California, USA

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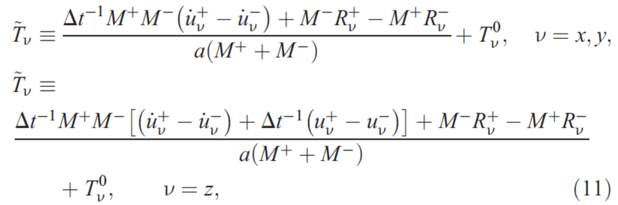
Division of Engineering and Applied Science, California Institute of Technology, Pasadena, California, USA

Received 2 May 2005; revised 5 October 2005; accepted 12 October 2005; published 23 December 2005.

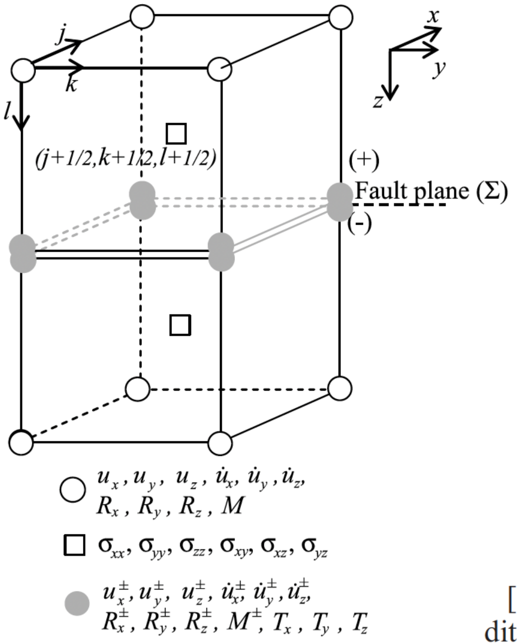
- TSN: a fault node is split into two halves (plus- & minus-side).

--Trial traction: enforce continuity of tangential v, and normal d.

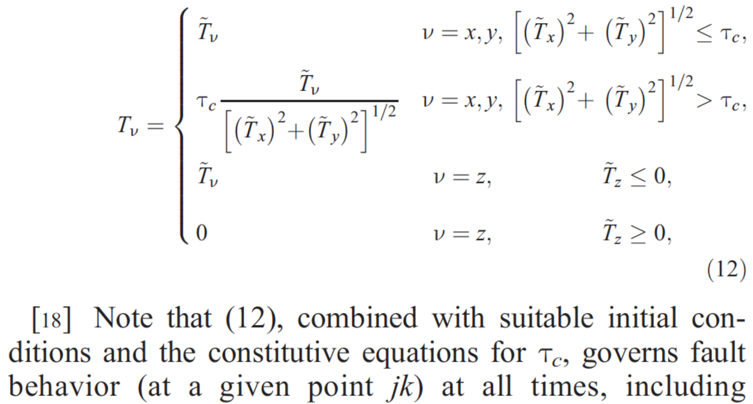


$$\tilde{T}_\nu \equiv \frac{\Delta t^{-1} M^+ M^- \left(\dot{u}_\nu^+ - \dot{u}_\nu^-\right) + M^- R_\nu^+ - M^+ R_\nu^-}{a(M^+ + M^-)} + T_\nu^0, \quad \nu = x, y,$$

$$\tilde{T}_\nu \equiv \frac{\Delta t^{-1} M^+ M^- \left[\left(\dot{u}_\nu^+ - \dot{u}_\nu^-\right) + \Delta t^{-1}\left(u_\nu^+ - u_\nu^-\right)\right] + M^- R_\nu^+ - M^+ R_\nu^-}{a(M^+ + M^-)} + T_\nu^0, \qquad \nu = z, \tag{11}$$



-- True traction components:

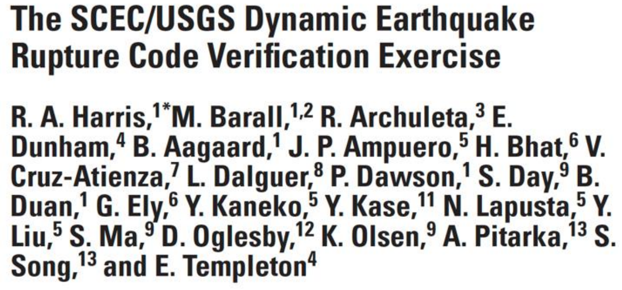


$$T_\nu = \begin{cases} \tilde{T}_\nu & \nu = x, y, \ \left[\left(\tilde{T}_x\right)^2 + \left(\tilde{T}_y\right)^2\right]^{1/2} \le \tau_c, \\ \tau_c \dfrac{\tilde{T}_\nu}{\left[\left(\tilde{T}_x\right)^2 + \left(\tilde{T}_y\right)^2\right]^{1/2}} & \nu = x, y, \ \left[\left(\tilde{T}_x\right)^2 + \left(\tilde{T}_y\right)^2\right]^{1/2} > \tau_c, \\ \tilde{T}_\nu & \nu = z, \quad \tilde{T}_z \le 0, \\ 0 & \nu = z, \quad \tilde{T}_z \ge 0, \end{cases} \tag{12}$$

[18] Note that (12), combined with suitable initial conditions and the constitutive equations for τ_c, governs fault behavior (at a given point *jk*) at all times, including

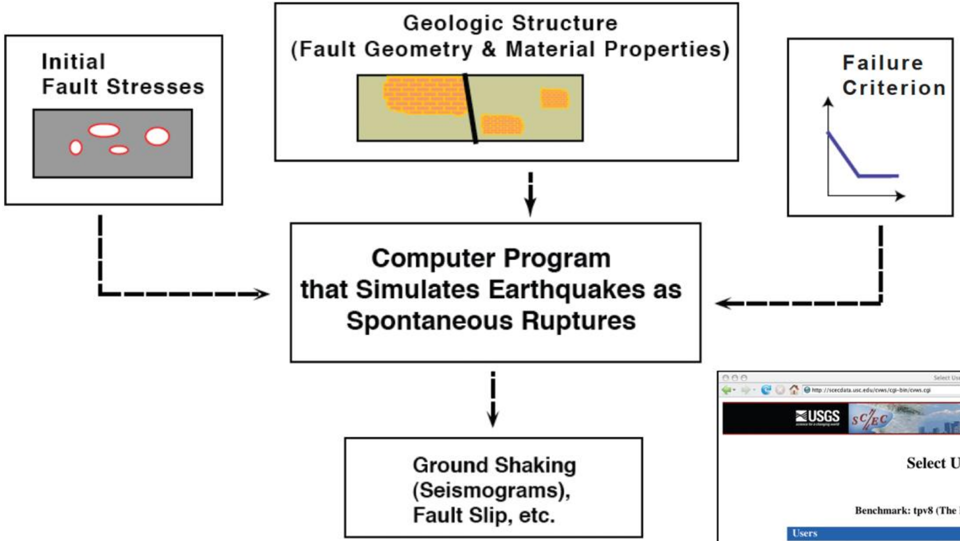
Classical Paper #4 on Methodology

- Harris et al (2009), The SCEC/USGS Dynamic Earthquake Rupture Code Verification Exercise, SRL.
- Code verification: no analytical solution.



The SCEC/USGS Dynamic Earthquake Rupture Code Verification Exercise

R. A. Harris,[1*] M. Barall,[1,2] R. Archuleta,[3] E. Dunham,[4] B. Aagaard,[1] J. P. Ampuero,[5] H. Bhat,[6] V. Cruz-Atienza,[7] L. Dalguer,[8] P. Dawson,[1] S. Day,[9] B. Duan,[1] G. Ely,[6] Y. Kaneko,[5] Y. Kase,[11] N. Lapusta,[5] Y. Liu,[5] S. Ma,[9] D. Oglesby,[12] K. Olsen,[9] A. Pitarka,[13] S. Song,[13] and E. Templeton[4]



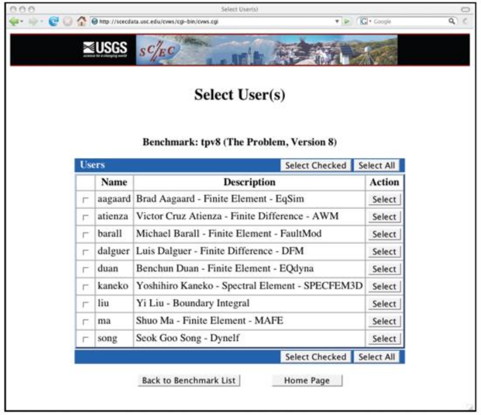
Initial Fault Stresses

Geologic Structure (Fault Geometry & Material Properties)

Failure Criterion

Computer Program that Simulates Earthquakes as Spontaneous Ruptures

Ground Shaking (Seismograms), Fault Slip, etc.



Select User(s)

Benchmark: tpv8 (The Problem, Version 8)

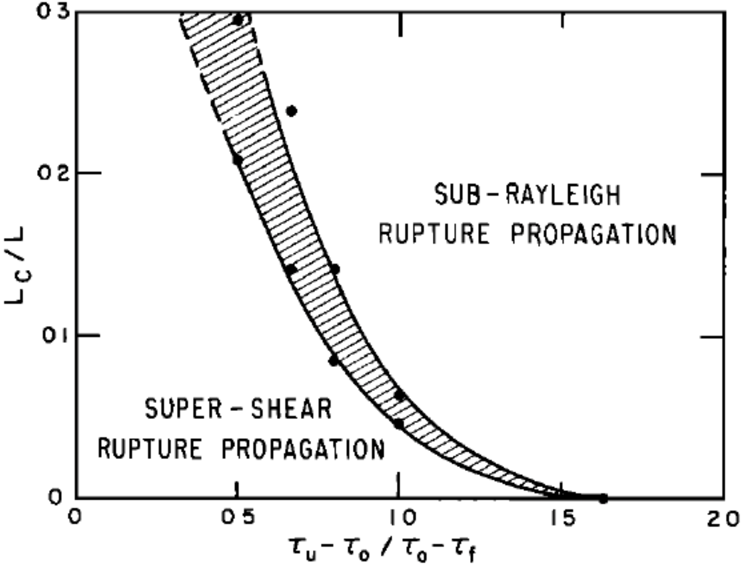
Users — Select Checked | Select All

	Name	Description	Action
☐	aagaard	Brad Aagaard - Finite Element - EqSim	Select
☐	atienza	Victor Cruz Atienza - Finite Difference - AWM	Select
☐	barall	Michael Barall - Finite Element - FaultMod	Select
☐	dalguer	Luis Dalguer - Finite Difference - DFM	Select
☐	duan	Benchun Duan - Finite Element - EQdyna	Select
☐	kaneko	Yoshihiro Kaneko - Spectral Element - SPECFEM3D	Select
☐	liu	Yi Liu - Boundary Integral	Select
☐	ma	Shuo Ma - Finite Element - MAFE	Select
☐	song	Seok Goo Song - Dynelf	Select

Important Application #1: Supershear Rupture

- Andrews (1976): 2D
- Dunham (2007, JGR): 3D

Max S: 1.19 (3D) vs. 1.77 (2D)



re velocity domains in parameter space. Shaded re-
nal. Vertical axis is ratio of critical length to crack
l axis is ratio of stress increase required to initiate slip
p.

Conditions governing the occurrence of supershear ruptures under slip-weakening friction

Eric M. Dunham[1]

Received 25 August 2006; revised 13 December 2006; accepted 27 March 2007; published 4 July 2007.

Important Application #2: Geometrically Complex Faults

- Harris and Day (1992, JGR): Stepover, 2D single-event
- Kame et al. (2003, JGR): Branch, 2D single-event
- Duan & Oglesby (2005, 2006, 2007, JGR): Bend, Stepover, Branch – 2D multicycle dynamics
- Lozos et al. (2011, BSSA): Stepover, 2D single-event, parameter space

Dynamics of Fault Interaction: Parallel Strike-Slip Faults

RUTH A. HARRIS

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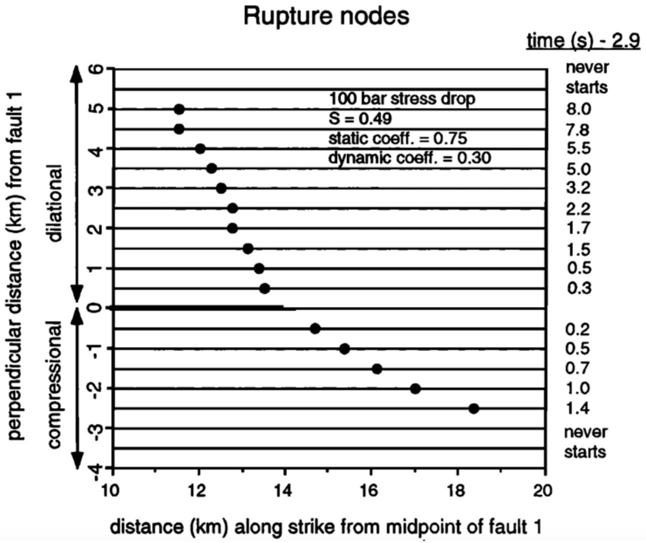
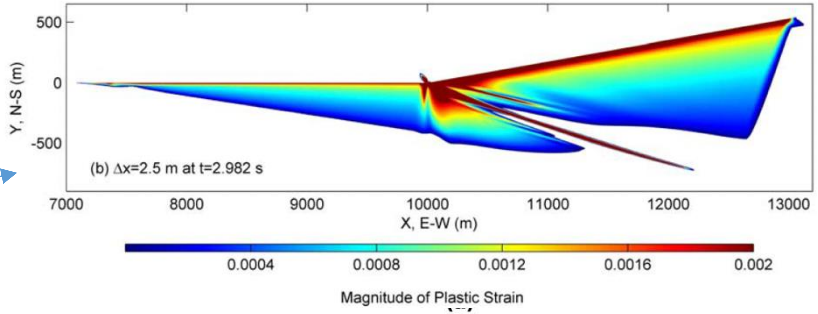
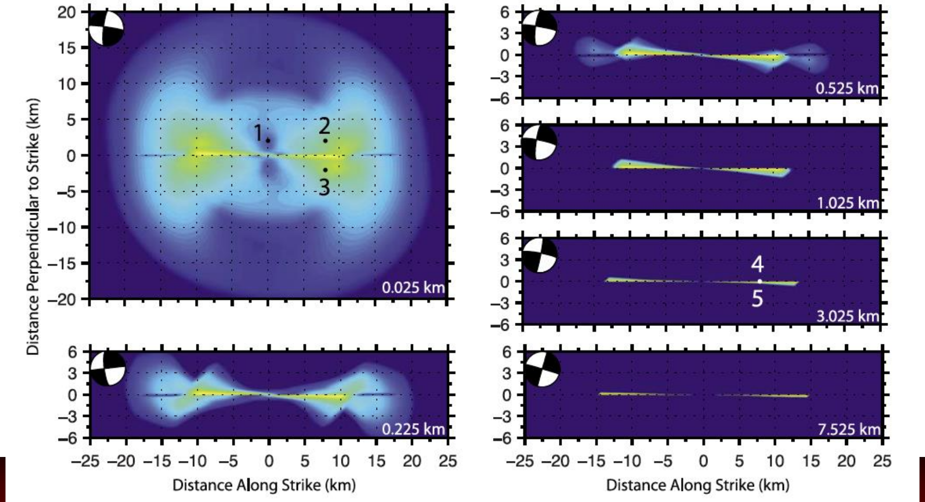


Important Application #3: Off-fault damage (plasticity)

- Andrews (2005, JGR): 2D
- Duan and Day (2008, JGR): 2D
- Ma and Andrews (2010, JGR): 3D





Important Application #4: Crack vs Pulse-like Ruptures

- Gabriel (2012, JGR): 2D models

The transition of dynamic rupture styles in elastic media under velocity-weakening friction

A.-A. Gabriel,[1] J.-P. Ampuero,[2] L. A. Dalguer,[1] and P. M. Mai[3]

Received 21 May 2012; revised 30 July 2012; accepted 5 August 2012; published 25 September

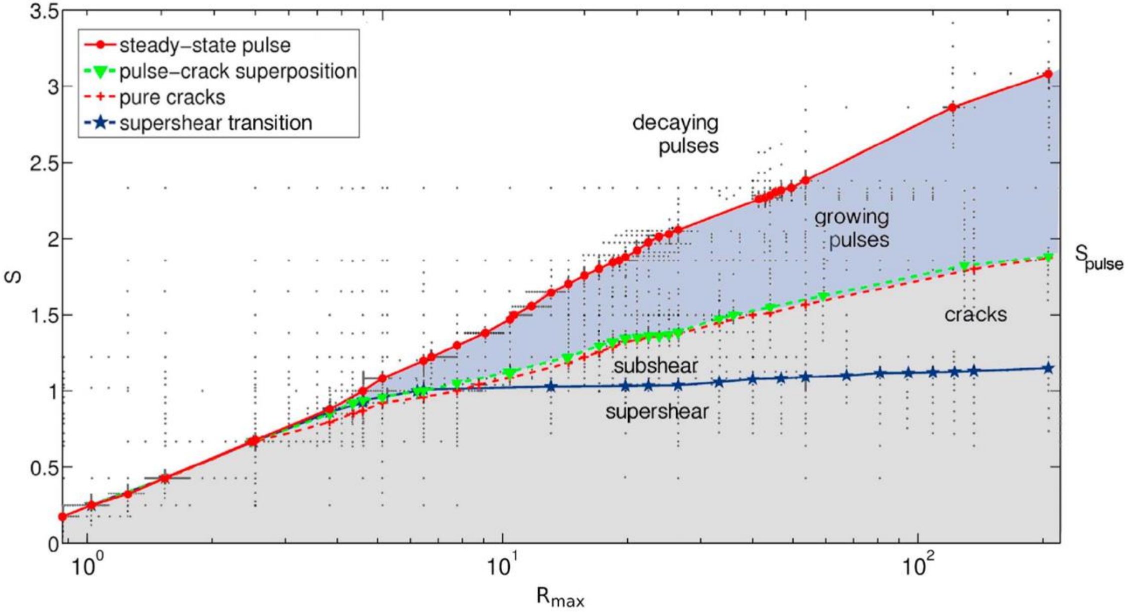
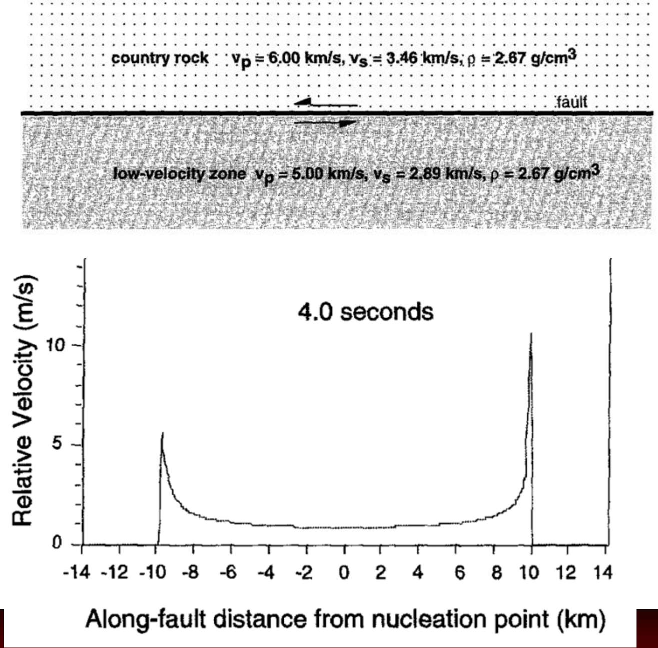
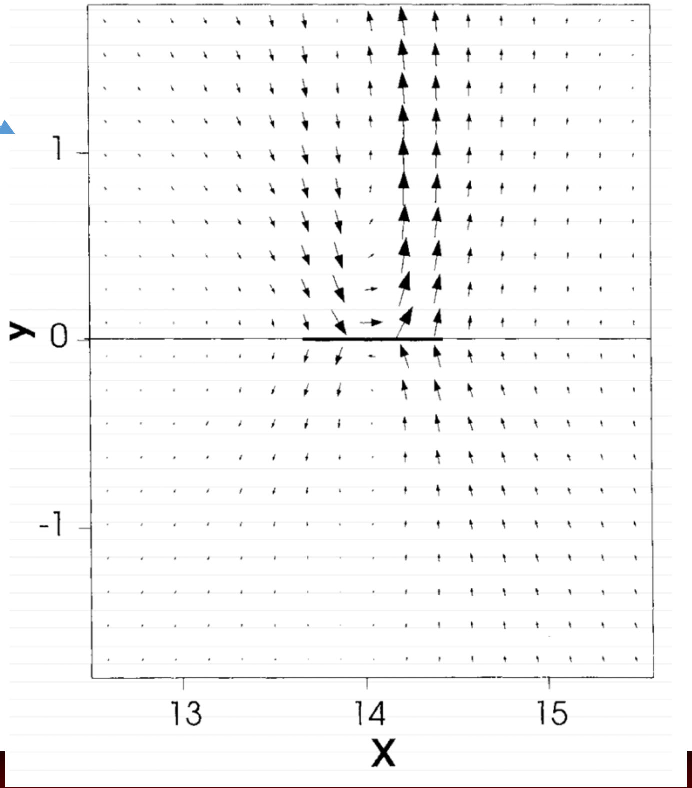


Figure 5. Summary of rupture styles as a function of S and R_{max} after nucleation with prescribed healing.

Important Application #5: Bimaterial Interface Rupture

- Andrews and Ben-Zion (1997, JGR)
- Harris and Day (1997, BSSA)





Important Application #6: Ruptures with strong v-weakening

- Andrews (2002, JGR)
- Noda et al. (2009, JGR)
- Dunham et al. (2011a, b, BSSA)

Earthquake Ruptures with Strongly Rate-Weakening Friction and Off-Fault Plasticity, Part 1: Planar Faults

by Eric M. Dunham, David Belanger, Lin Cong, and Jeremy E. Kozdon

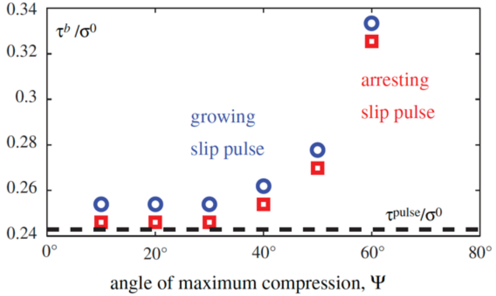
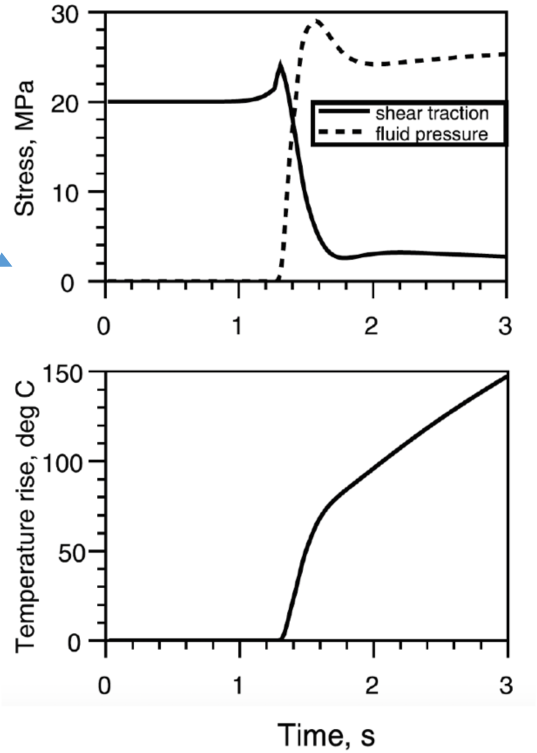


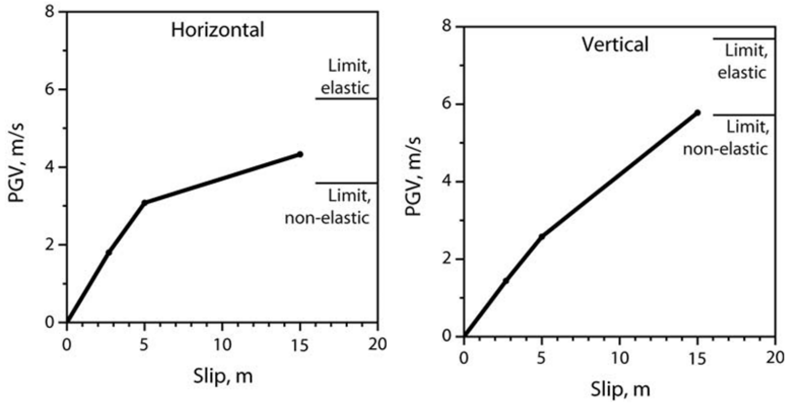
Figure 6. Minimum background stress, τ^b, required for self-sustaining rupture propagation as a function of prestress orientation,

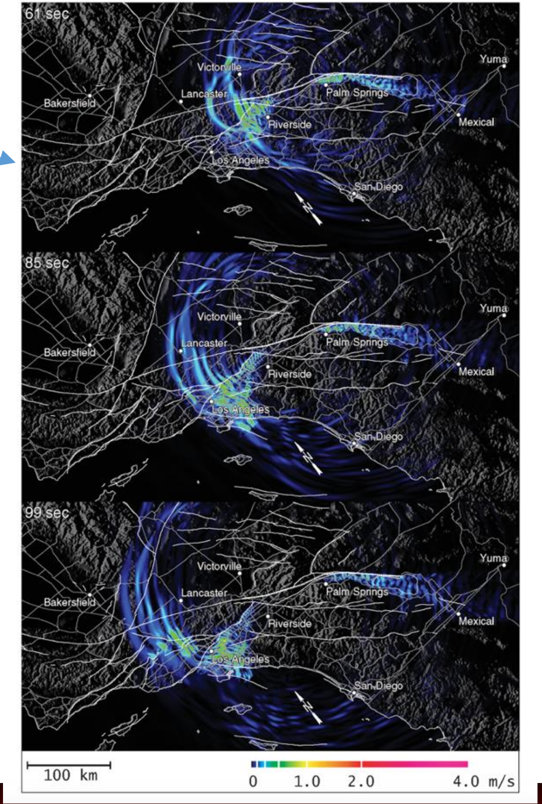


Important Application #7: Ground Motion Simulation

- Olsen et al. (2008, BSSA): TeraShake2
- Andrews (2007, BSSA): Physical Limits
- …

- PGV limits from dynamic models





A future direction for dynamic rupture modeling: Duan’s view

2. PUTTING 3D DYNAMIC RUPTURE MODELING IN THE CONTEXT OF EARTHQUAKE CYCLE SIMULATIONS

Interlude: Single-event dynamic rupture vs multicycle dynamic rupture

- Single-event dynamics:
  - Methodology development: more physics …
  - A lot of applications to explore EQ source physics:
  - Main restriction: assumed initial stresses
- Earthquake cycles simulations with dynamic rupture included:
  - Handle the above restriction for dynamic rupture modeling
  - Stresses evolve spontaneously and are consistent with fault geometry and rupture history: different rupture behaviors, typical events etc.
  - Explore various slip behaviors (EQs, SSEs, …) and their interactions
  - Assimilate a variety of data; Explore physics; Conduct physics-based seismic hazard analysis, including GM simulation/prediction …

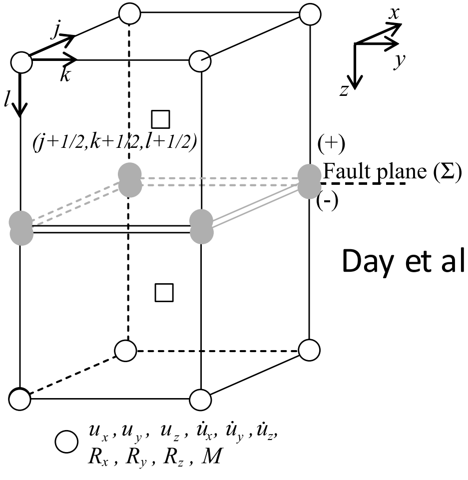
EQdyna: an explicit FEM method, from a dynamic rupture code to a dynamic earthquake simulator.

- EQdyna before 2020: an explicit FEM code for dynamic rupture only
- Implement rate- & State-dependent friction into EQdyna:
  - Luo and Duan (2018)
- Adopt a dynamic relaxation scheme to EQdyna: solve static problems
  - Luo, Duan, & Liu (2020)
  - Simulate the quasi-static processes: nucleation, post- & inter-seismic
- EQdyna now: a dynamic earthquake simulator
  - For earthquake cycle simulations with dynamic rupture included.
  - Can simulate earthquake behaviors on geometrically complex faults embedded in heterogeneous geological structure over many cycles.
  - Can capture both seismic and aseismic slip: explore their interactions.

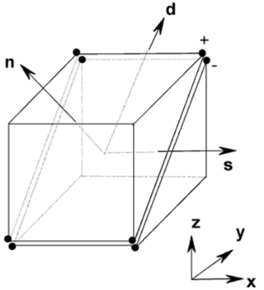
Modeling Fractures in EQdyna: TSN (traction-at-split-node) method

- A fracture is specified as a surface of split nodes: e.g., fault plane in the figure.
- A discontinuity in the displacement vector is permitted across the surface.
    - ✓**Shear fracture**: tangential displacement discontinuity. **Fault friction**.
    - ✓**Tensile/Opening fracture**: normal displacement discontinuity. **Hydraulic fracturing**.
- Coupling across the fracture is accomplished by specifying surface traction.

$$\mathbf{a}_n = \mathbf{M}^{-1}(\mathbf{F}_n - \mathbf{K}(\mathbf{u}_n + q\mathbf{v}_n) + \mathbf{H}_n \pm \mathbf{R}_n).$$



Day et al. (2005)



Duan (2010)

HPC version of EQdyna: Hybrid OpenMP/MPI

- *Scaling tests on SCEC TPV 11 and 210*

Wu, Duan et al. (2011)

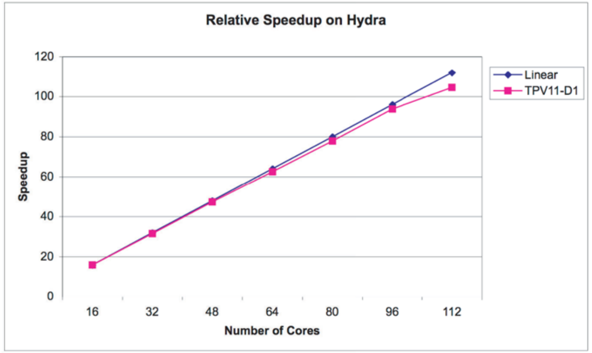
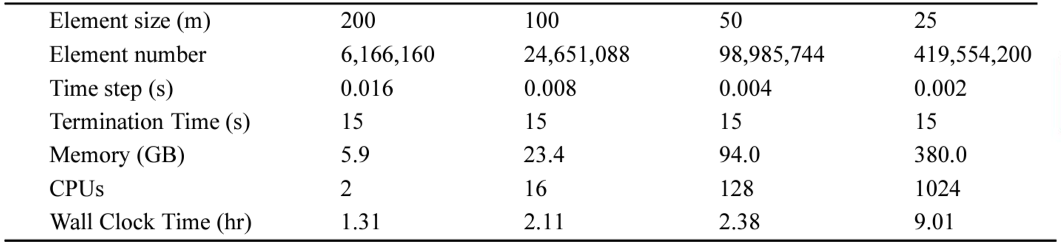


Figure 18. Speedup of the hybrid implementation on Hydra

Duan (2012a)

Table 5.2 Model sizes and computational resources used in the convergence test of a benchmark problem



Element size (m)	200	100	50	25
Element number	6,166,160	24,651,088	98,985,744	419,554,200
Time step (s)	0.016	0.008	0.004	0.002
Termination Time (s)	15	15	15	15
Memory (GB)	5.9	23.4	94.0	380.0
CPUs	2	16	128	1024
Wall Clock Time (hr)	1.31	2.11	2.38	9.01

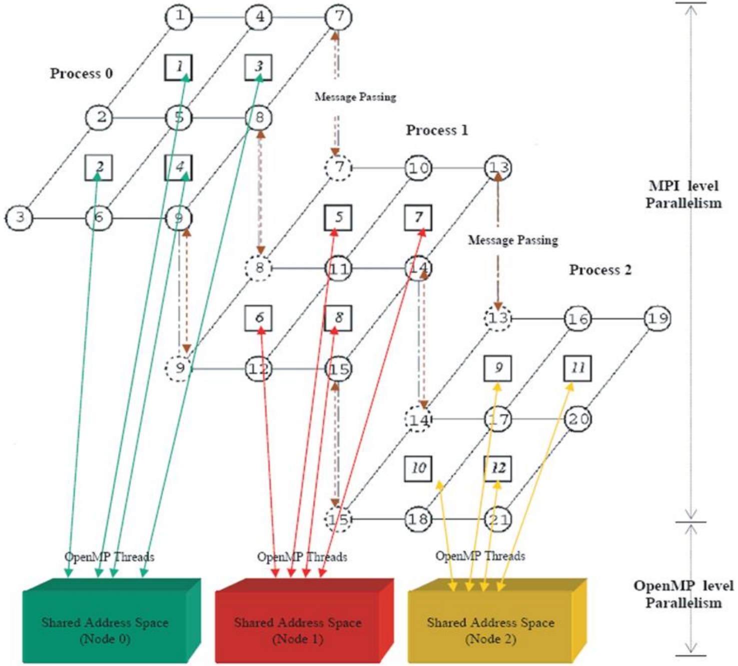
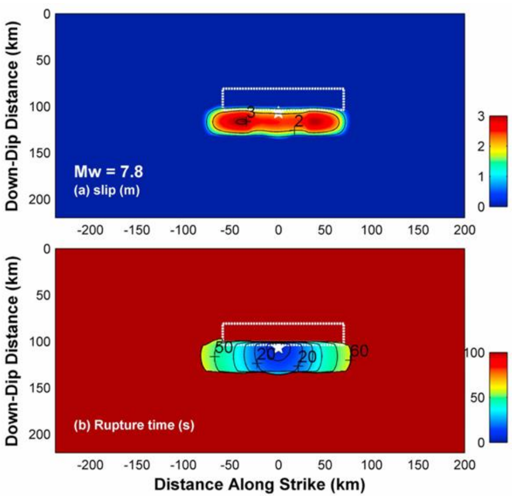


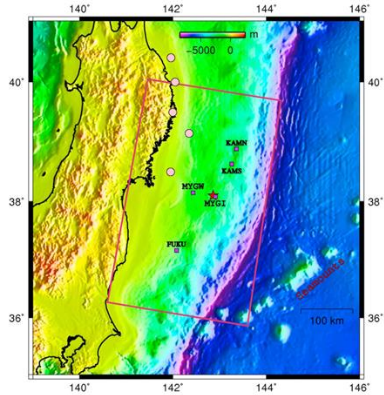
Figure 13. Parallelism at MPI and OpenMP levels within one timestep

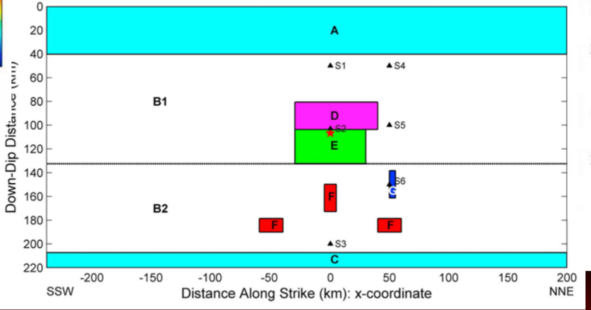
EQdyna: Dynamic rupture modeling – an example

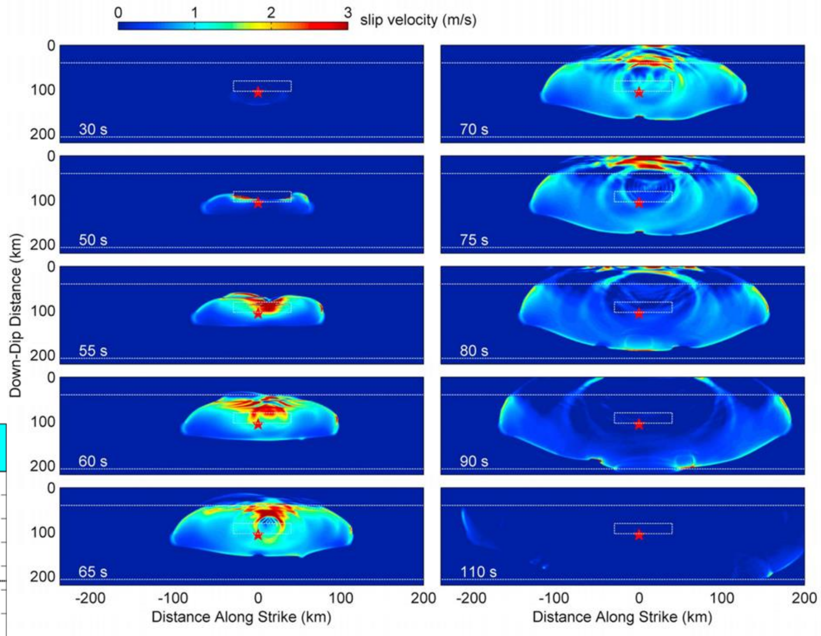
- **A dynamic rupture model of the 2011 Mw 9.0 Tohoku EQ**: Duan (2012)
  Roles of a possible subducting seamount



A longer & stronger seamount: not fail, Mw 7.8

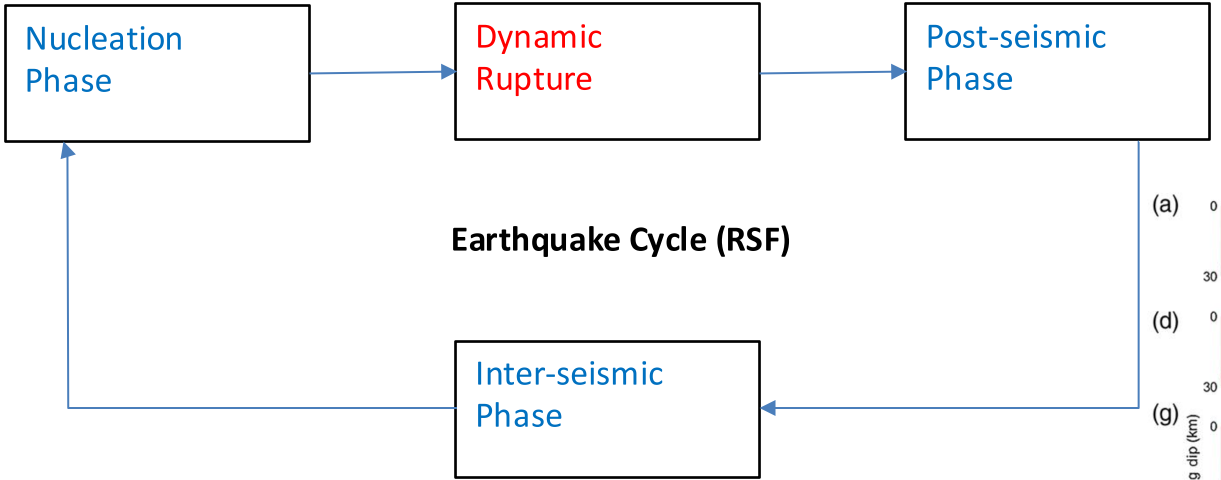






Seamount’s failure: Mw 9.0

EQdyna-based Dynamic Earthquake Simulator



Nucleation Phase

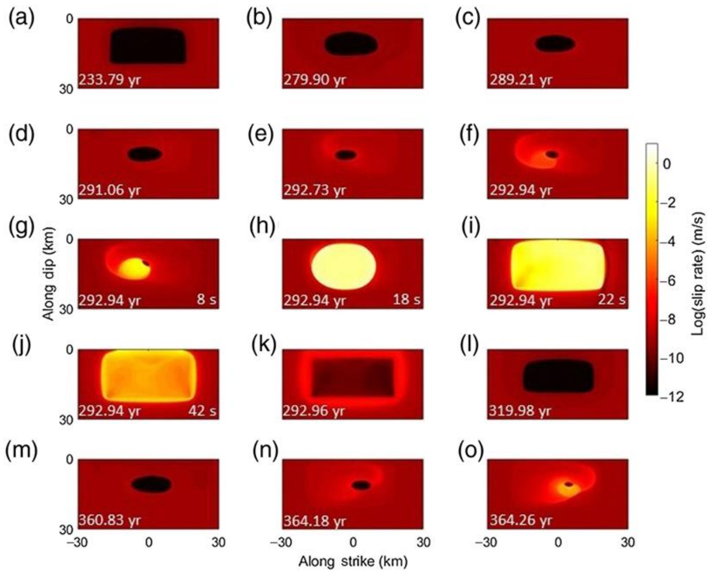
Dynamic Rupture

Post-seismic Phase

Inter-seismic Phase

Earthquake Cycle (RSF)

Luo et al. (2020)



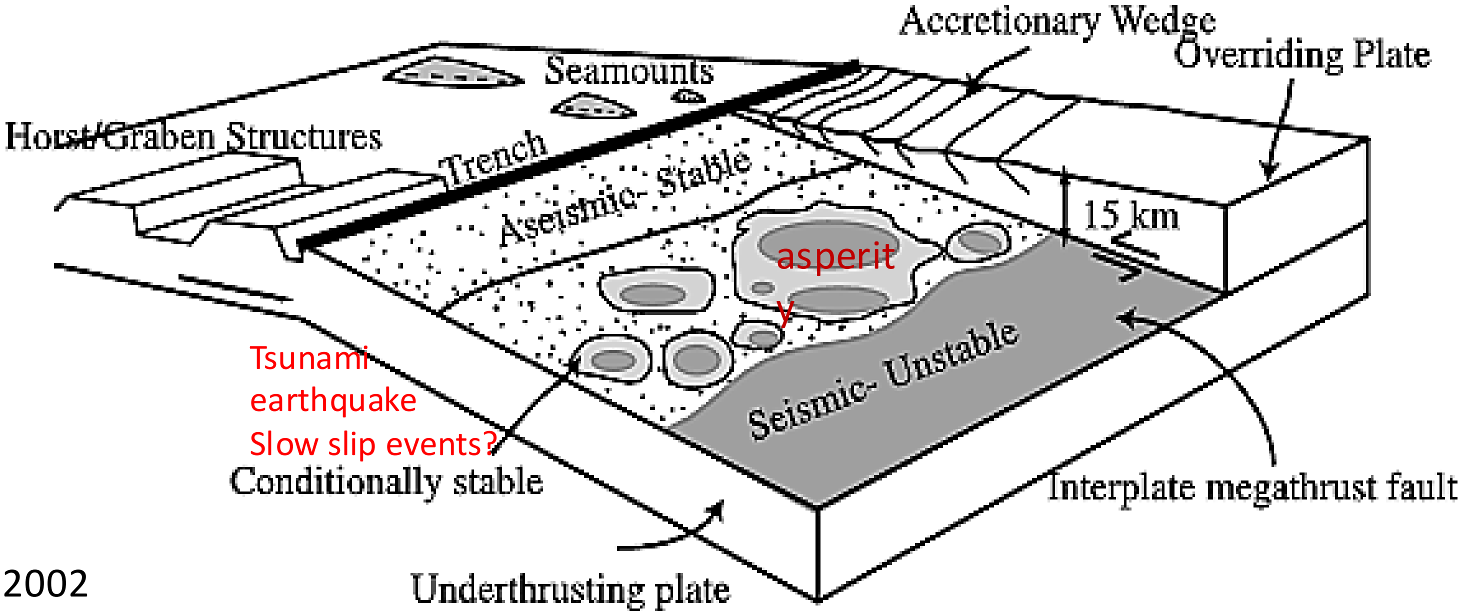
- Dynamic processes: directly use EQdyna
- Quasi-static processes: EQdyna with DR (Dynamic Relaxation)

Applications of the Dynamic Simulator EQdyna: Putting dynamic rupture within earthquake cycles

- Various slip behaviors and their interactions along subduction zones over earthquake cycles.
- Earthquake rupture behaviors (patterns, extents, recurrence etc) of geometrically complex faults & real fault systems such as SAF over many cycles.
- Recent, complex large earthquakes in the context of rupture history of a fault system.
- Ground motion simulations from typical earthquakes over many cycles.
- ……

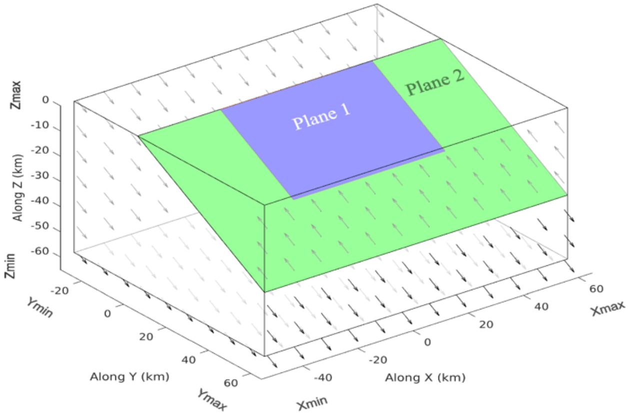
Example #1: Tsunami earthquake generation Meng et al. (2022)

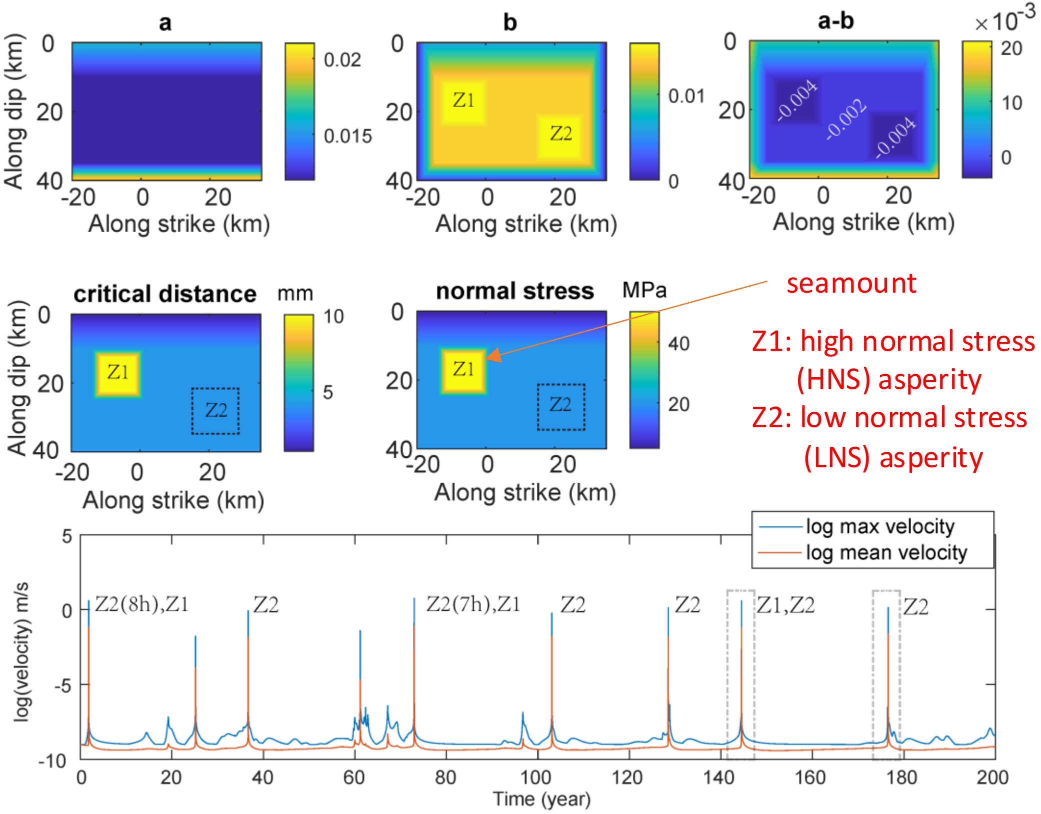
- A conceptual model as tsunami earthquake mechanism



Bilek and Lay, 2002

Using the simulator to explore frictional control on tsunami EQ generation





seamount

Z1: high normal stress (HNS) asperity

Z2: low normal stress (LNS) asperity

On-fault analyses for dynamic ruptures in Model 1

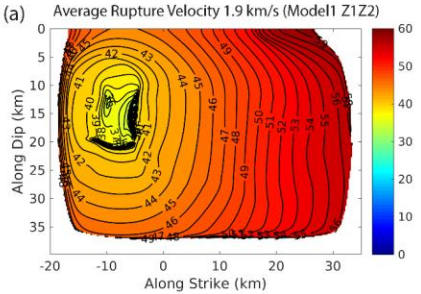
Rupture time contours

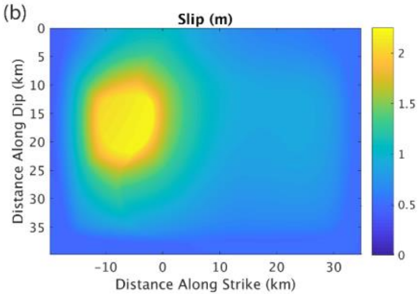
Slip distribution

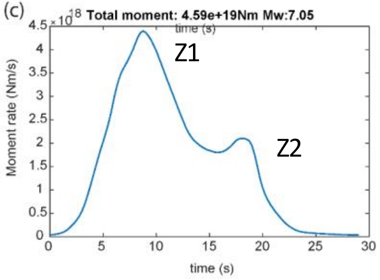
Moment rate

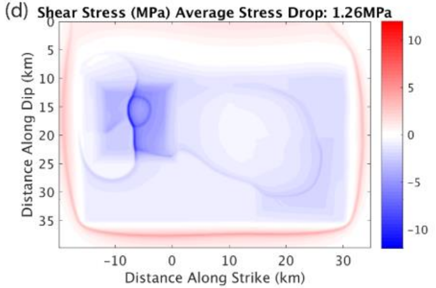
Stress change distribution

Nucleated from Z1



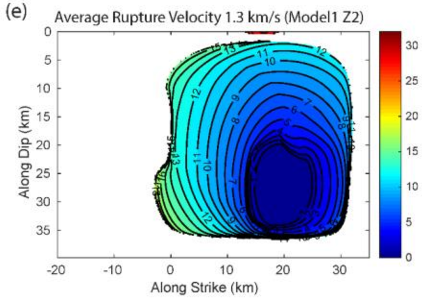


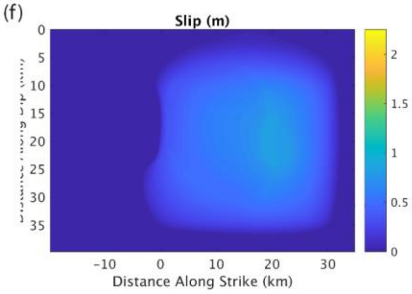


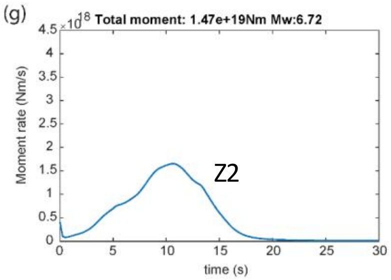


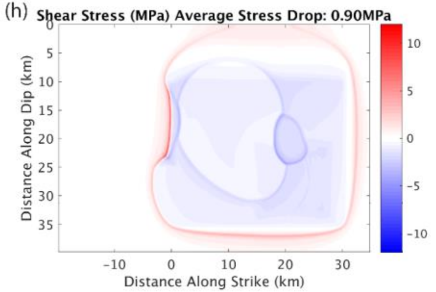
rv= 1.9 km/s

Nucleated from Z2





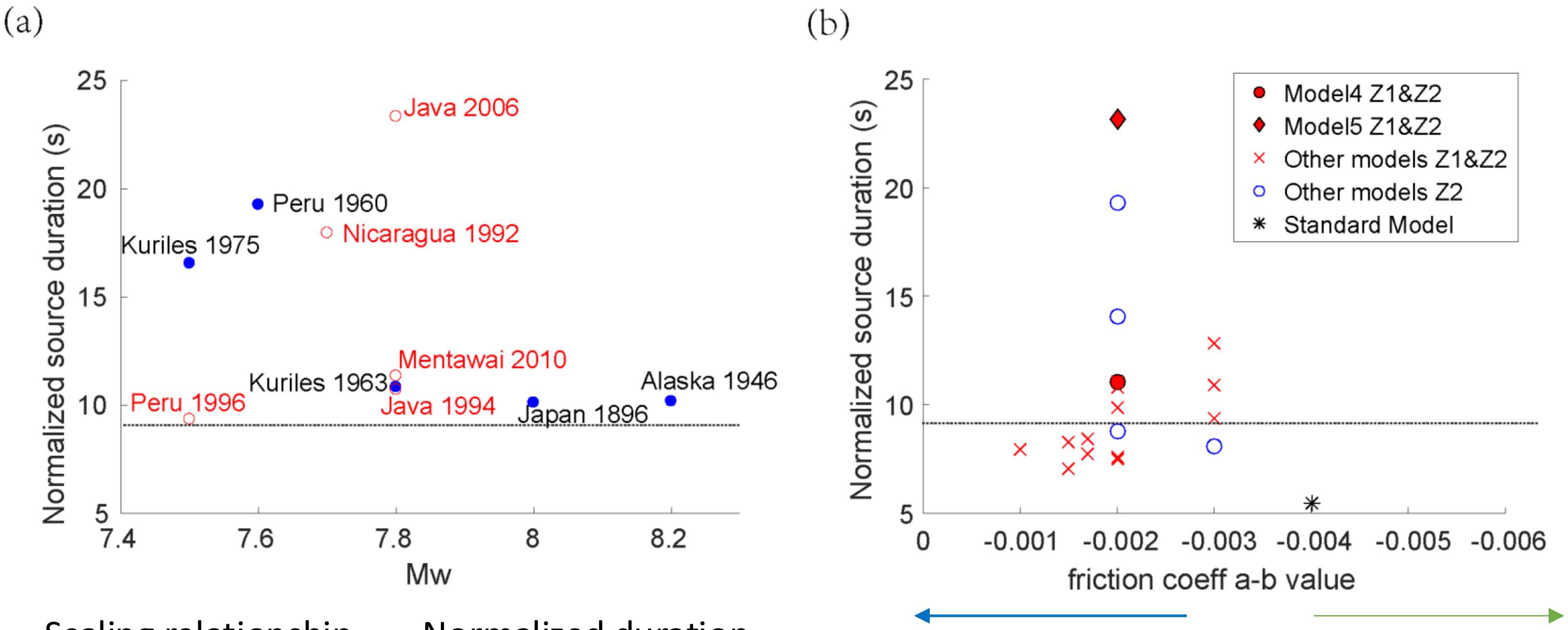




rv= 1.3 km/s

$$M_0 = \mu D S$$

Observed vs Simulated tsunami earthquakes: source durations



Weaker velocity weakening

Stronger velocity weakening

Scaling relationship

T ~ $\sqrt[3]{M_0}$

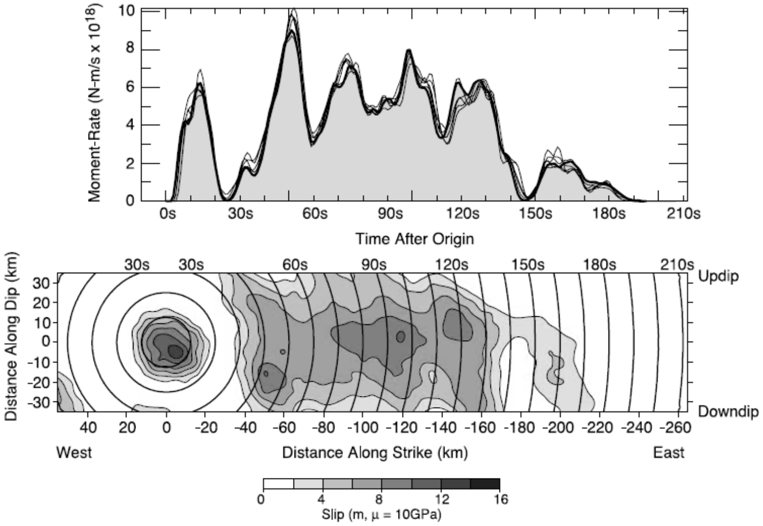
Normalized duration

$\frac{T}{\sqrt[3]{M_0}}$ to M_w=6.0

Conclusions on tsunami EQ generation

- The conceptual model (asperities + conditionally stable zone) works well for generating tsunami earthquakes, of characteristics of slow rupture velocity, long normalized duration and spectrum depleted in high frequency.
- The level of velocity-weakening of the conditionally stable zone is critical to sustain rupture at slow speeds.
- High normal stress asperities (seamounts) act as barriers in small earthquakes while as asperities in large cascading events.
- Low normal stress asperities are relatively easy to be ruptured in a cascade fashion

Java 2006 tsunami earthquake



Ammon et al. 2006

Example #2: Interaction between SSEs and megathrust earthquakes Meng & Duan (2022)

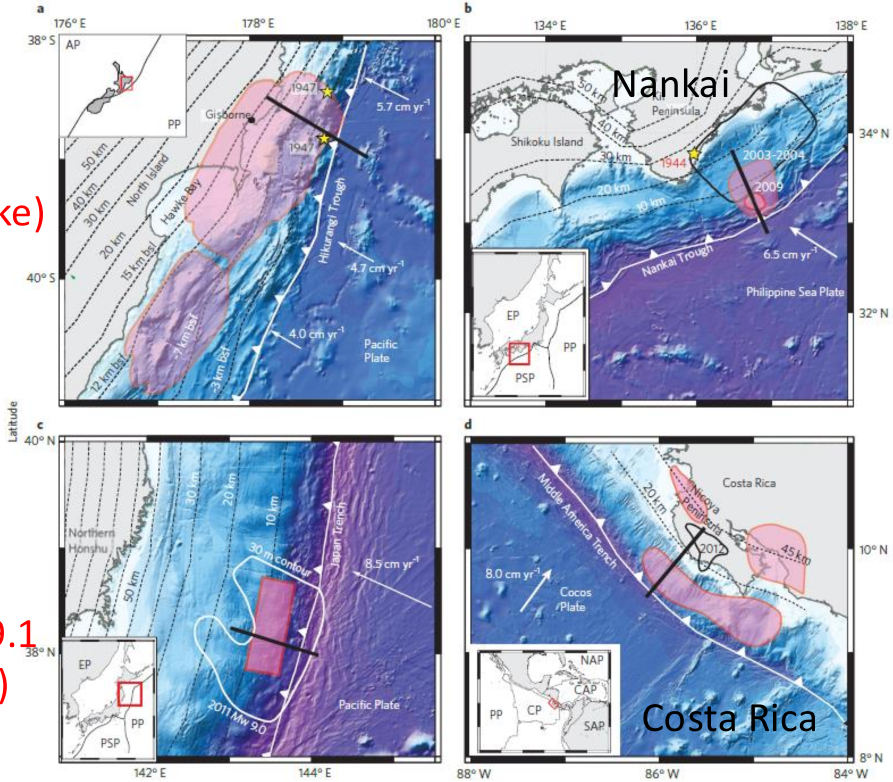
- SSEs = Slow Slip Events:

widely observed along subduction zones

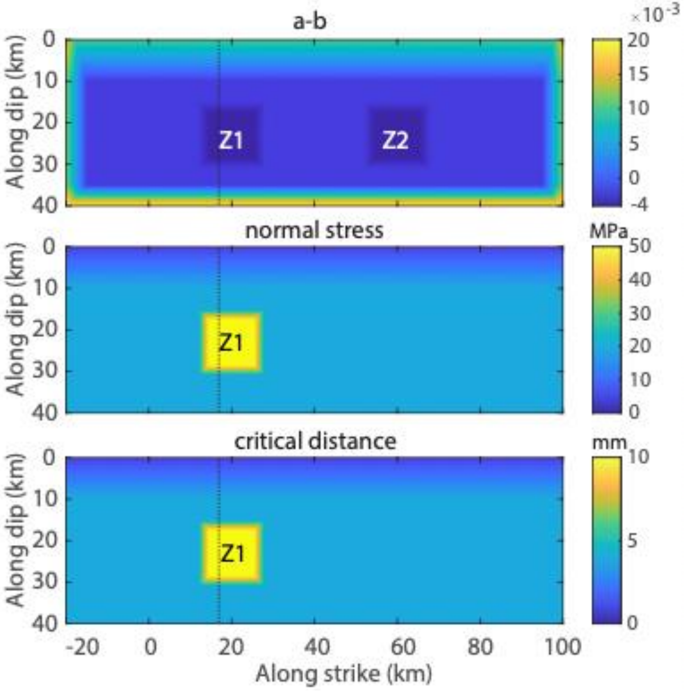
- Observations suggest possible interactions with megathrust earthquakes.

Hikurangi (1947 tsunami earthquake)

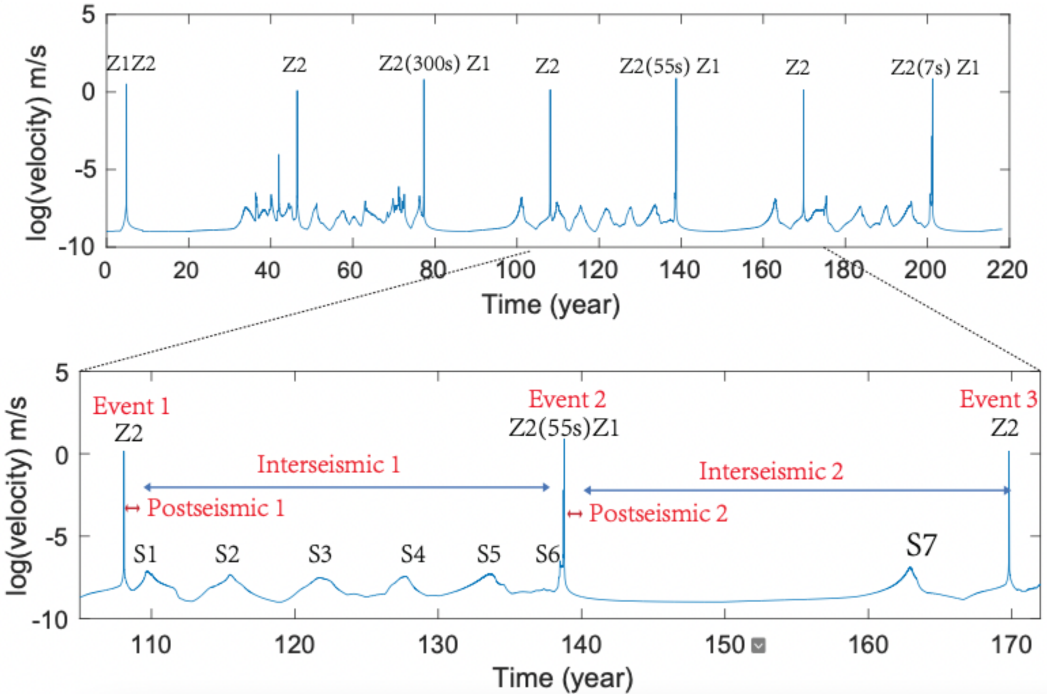
Tohoku (2011 Mw 9.1 earthquake)



Model

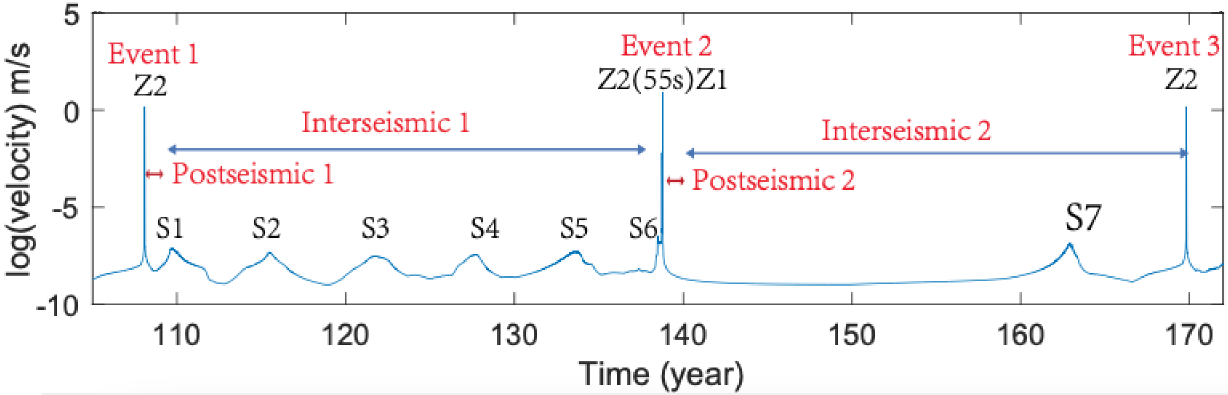


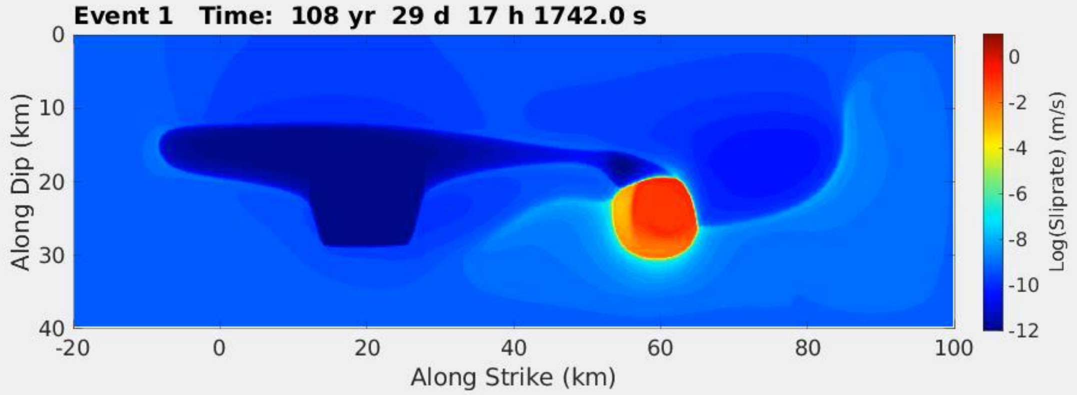
1 HNS (Z1) and 1 LNS (Z2) zone in
conditionally stable zone



6 Slow slip events (SSEs) in interseismic 1
1 SSE in interseismic 2

Movie starting from Event 1 ending at Event 3





Conclusions on interactions between EQs and SSEs

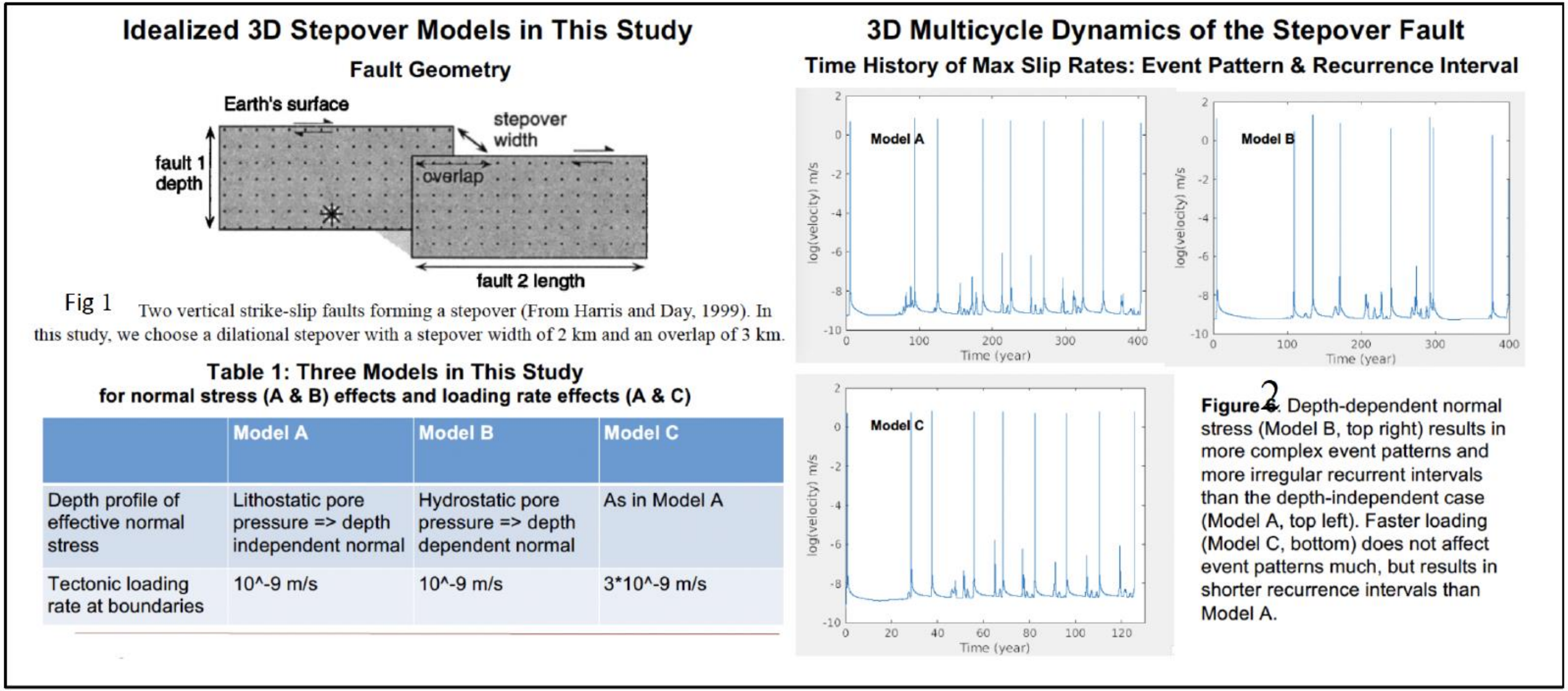
- Small earthquakes (Type I events) are preceded by fewer SSEs.

- Large earthquake (Type II events) are preceded by many SSEs.

- The interseismic coupling degree is low preceding a Type II earthquake due to active SSEs, and is high preceding Type I earthquake due to much fewer preceding SSEs.

Types	Examples	Magnitude (Mw)	No. of preceding SSEs	No. of following SSEs	Ruptured asperities	Recurrence interval (years)	Ruptured Length (km)	Average rupture speed (km/s)
Type I	Event 1 Event 3	~ 7.1	1	6	Z2	~ 60	~ 70	~ 1.5
Type II	Event 2	~ 7.3	6 (S1-S6)	1 (S7)	Z1 & Z2	~ 60	~ 110	~ 0.7

Example #3: 3D multicycle dynamics of stepover faults Duan (2023, AGU)



Idealized 3D Stepover Models in This Study

Fault Geometry

Fig 1 Two vertical strike-slip faults forming a stepover (From Harris and Day, 1999). In this study, we choose a dilational stepover with a stepover width of 2 km and an overlap of 3 km.

Table 1: Three Models in This Study
for normal stress (A & B) effects and loading rate effects (A & C)

	Model A	Model B	Model C
Depth profile of effective normal stress	Lithostatic pore pressure => depth independent normal	Hydrostatic pore pressure => depth dependent normal	As in Model A
Tectonic loading rate at boundaries	10^-9 m/s	10^-9 m/s	3*10^-9 m/s

3D Multicycle Dynamics of the Stepover Fault

Time History of Max Slip Rates: Event Pattern & Recurrence Interval

Figure ~~6~~2. Depth-dependent normal stress (Model B, top right) results in more complex event patterns and more irregular recurrent intervals than the depth-independent case (Model A, top left). Faster loading (Model C, bottom) does not affect event patterns much, but results in shorter recurrence intervals than Model A.

Final Slip & Hypocenter Location of Model B: Rupture Pattern

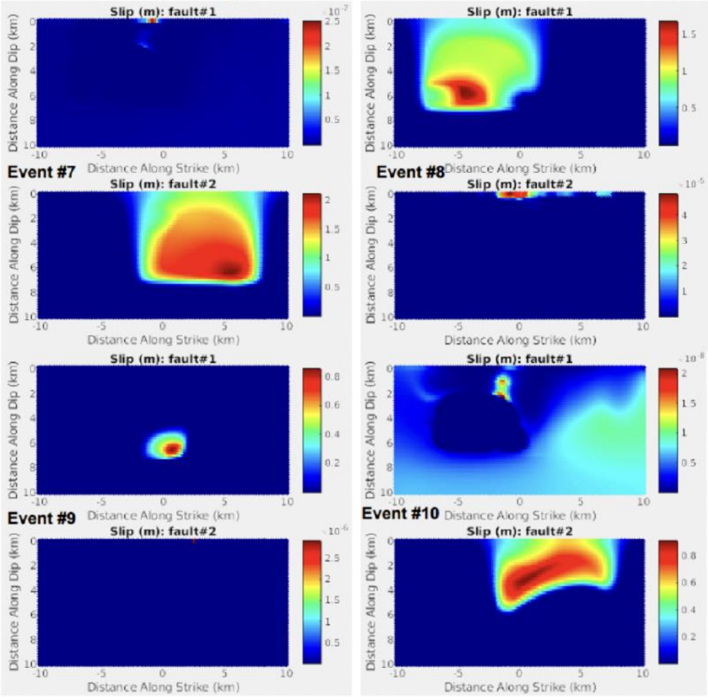
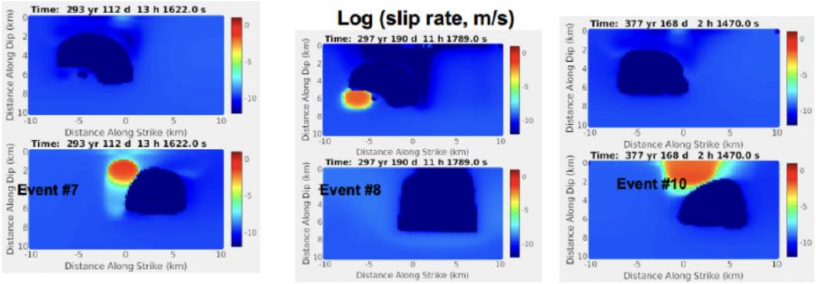


Figure 3 (left). Final Slip of Events #7, 8, 9, 10 in the multicycle dynamics of Model B, showing different rupture behaviors.

Figure 4 (bottom). Nucleation (hypocenter) location of Events #7, 8, and 10 in Model B, spontaneously determined by previous ruptures.



Example #4: Fault-dip effects on EQ ruptures along the Mojave segment of SAF Bordbar et al. (2024, SCEC), on-going

- Background: Paleoseismic observations (Bemis et al., 2021)
  - o Fewer earthquakes along the straight Mojave segment than the surrounding

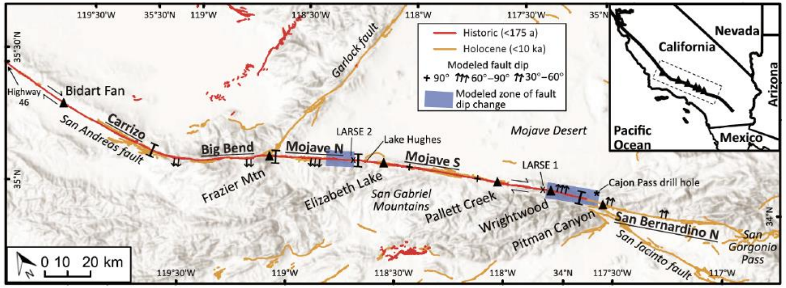
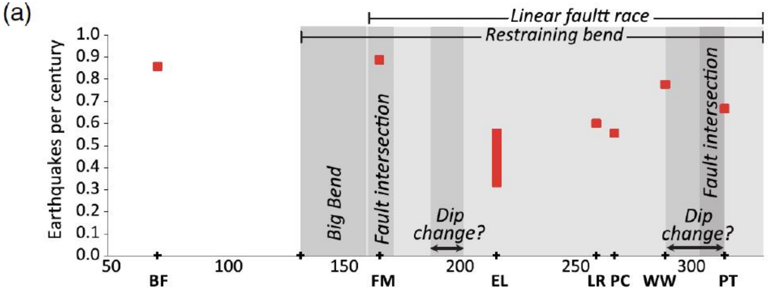


Figure 2.The southern San Andreas fault (SSAF) system and the locations of paleoseismic sites (black triangles) with dipping fault geometry from Fuis et al. (2012). (from Bemis et al., 2021).



Example #4: Fault-dip effects on EQ ruptures along the Mojave segment of SAF Bordbar et al. (2024, SCEC), on-going

DATA AND MESH MODELS

Fig. 1 (a) Map view of vertical SAF model surface and (b) dipping SAF model surface

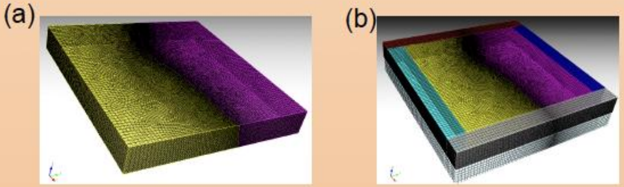


Fig. 2 Mesh model of the San Andreas Fault created using CUBIT software for (a)Quasi static phase and (b) Dynamic phase.

MODEL PARAMETERS ON THE FAULT

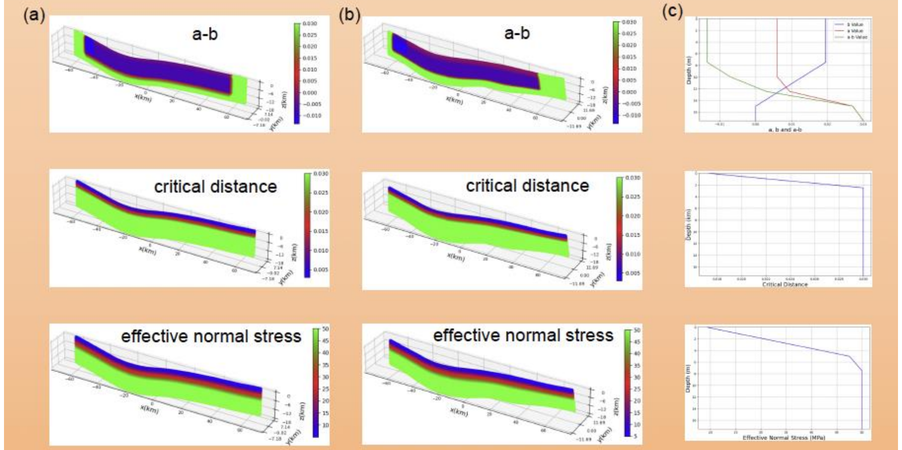


Fig.3 Distributions of a–b in the rate-and-state friction law, critical distance effective normal stress (a) for vertical dipping and (b)) dip-varying fault geometry , and (c) the profiles of each parameter..

Challenges & Strategies

3. CONCLUDING REMARKS

Challenging problems need powerful tools

- 3D earthquake cycle simulations with coseismic dynamic rupture included are are very challenging due to **a large range of scales** in
  - ➢Time: from seconds to thousand years
  - ➢Space: from meters to thousand kilometers
- Integrating with observations for real case studies requires **handling complexities in models**
  - ➢Mesh generation for complex fault and velocity structures
  - ➢Largely hexahedra elements with degenerated wedges/tetrahedra

Strategies

- Further **parallelizing** EQdyna
  - Scale it to hundreds of thousands of CPUs
  - Implement GPU accelerators into it
  - Other emerging techniques
- Integrating the simulator with a 3rd-party **mesh generator**
  - Currently, mesh generation is integrated with the solver: good for MPI parallelization, but it is challenging to create complex mesh with (largely) hexahedra elements.

Computation resource used in current studies

Texas A&M High Performance Research Computing (https://hprc.tamu.edu)

Grace cluster

Software: EQdyna (Dynamic Earthquake simulator)

Job size: Elements: 30,750,300
CPU cores 600
Memory 300 GB
Running time 62 hours for about 10 earthquake cycles

Thank you for your attention !