Magnitude-weighted likelihood scores for earthquake forecasts.

1. Background.

2. Proposed weighted measures.

3. Application to CSEP models.

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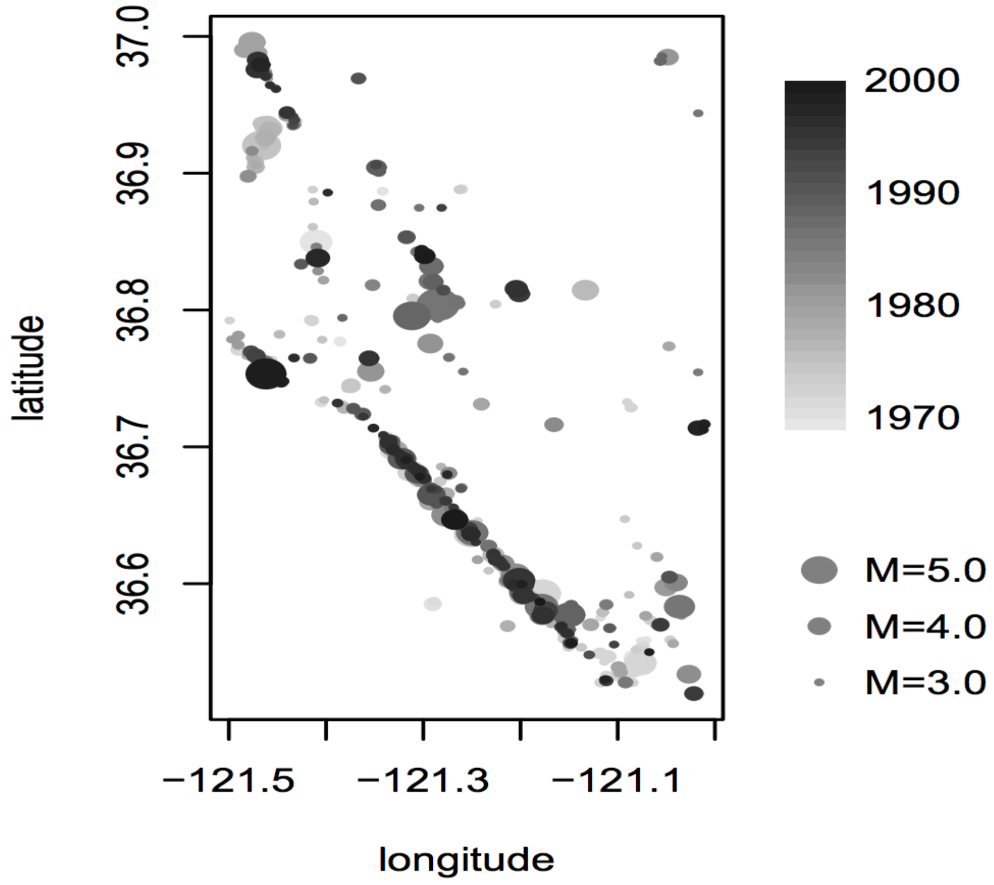

1. Background.

The state of the art is of very limited use.

ETAS models have been the best-fitting models in earthquake forecasting experiments

(Schorlemmer et al. 2010, Zechar et al. 2013, Bayona et al. 2022).

However, ETAS models are not very useful for forecasting large earthquakes.

ETAS models are useful for describing the spatial-temporal distribution of aftershocks, or perhaps as a null model to which alternative models might be compared.

However, for forecasting the largest events, ETAS is little better than a simple homogeneous Poisson model.

"Is ETAS the end?"

... Danijel Schorlemmer

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Earthquakes Cannot Be Predicted

R. Geller, D. Jackson, Y. Kagan, F. Mulargia Published in Science 14 March 1997 • Geology, Physics

2. Proposed weighted measures.

In fitting parameters to earthquake occurrence models such as ETAS, and especially for evaluating their goodness-of-fit, more weight should be assigned to the largest events.

Purposes of earthquake occurrence models:

- -- Prediction of largest events, public safety.
- -- Long-term forecasting, insurance, building codes.
- -- Physical principles, scaling from small to large events.

Weighting makes sense for all these purposes.

Weight using Damage, MMI, Magnitude, Energy?

2. Proposed weighted measures.

Weight using Damage, MMI, Magnitude, Energy?

$E = 10^{4.8 + 1.5\ Mw}$ (Kanamori, 1977).

Instead of $\sum \log(\lambda_i) - \int \lambda(t,x,y,m)\ dt\ dx\ dy\ dm$,

use $\sum \log(\lambda_i)\ 10^{4.8 + 1.5\ m_i} - \int \lambda(t,x,y,m)\ 10^{4.8 + 1.5m}\ dt\ dx\ dy\ dm$.

2. Proposed weighted measures.

In addition, one could consider more direct scores emphasizing the largest magnitudes, such as

$Q = \text{mean}(\lambda_i(t,x,y): m_i > m^{[.95]}) / \text{mean}(\lambda)$, (Schoenberg and Schorlemmer 2024)

where $m^{[.95]}$ is the 95th percentile of magnitudes in the dataset.

The numerator is the mean of λ over all magnitudes, but only at the locations and times where the largest 5% of events occurred.

The higher the value of the quotient, Q, the better the model is forecasting the largest events.

For a homogeneous Poisson model, Q will be close to 1.

Models adequately accounting for spatial inhomogeneity will have $Q > 1$.

With two competing models, consider the difference between *log-likelihoods*, in each pixel. The results are called ***deviance residuals*** (Clements et al. 2011), ~ resids from gen. linear models.

$$R_D(B_i) = \sum_{i:(t_i,x_i,y_i)\in B_i} \log(\hat{\lambda}_1(t_i,x_i,y_i)) - \int_{B_i} \hat{\lambda}_1(t,x,y)\,dt\,dx\,dy - \left(\sum_{i:(t_i,x_i,y_i)\in B_i} \log(\hat{\lambda}_2(t_i,x_i,y_i)) - \int_{B_i} \hat{\lambda}_2(t,x,y)\,dt\,dx\,dy \right).$$

Instead of $\sum \log(\lambda_i) - \int \lambda(t,x,y,m)\,dt\,dx\,dy$,

use $\sum \log(\lambda_i)\, 10^{4.8 + 1.5\, m_i} - \int \lambda(t,x,y,m)\, 10^{4.8 + 1.5m}\, dt\,dx\,dy\,dm$.

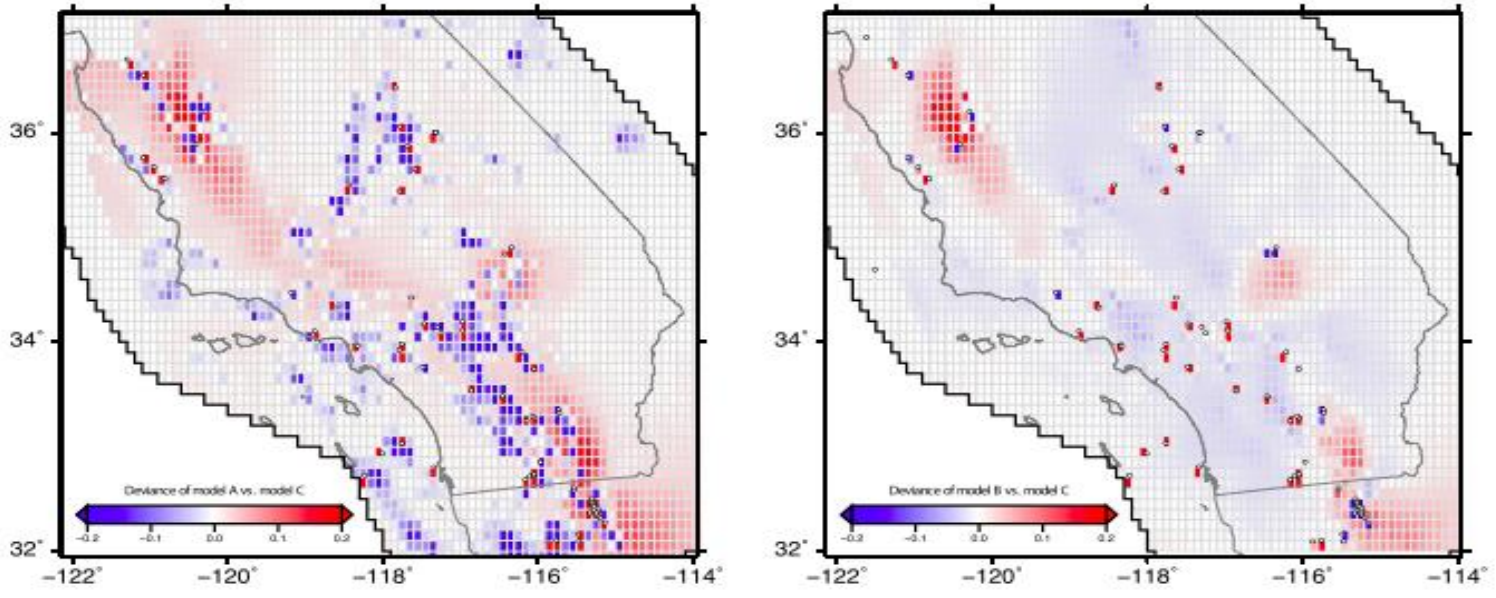


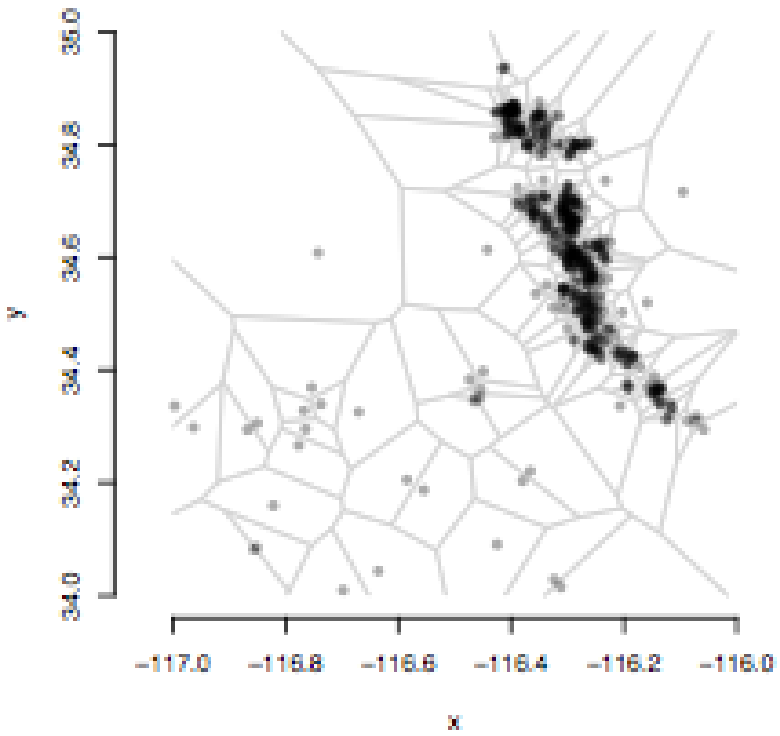
FIG. 4. *Left panel* (a): *deviance residuals for model A versus C. Sum of deviance residuals is* 86.427. *Right panel* (b): *deviance residuals for model B versus C. Sum of deviance residuals is* −7.468.

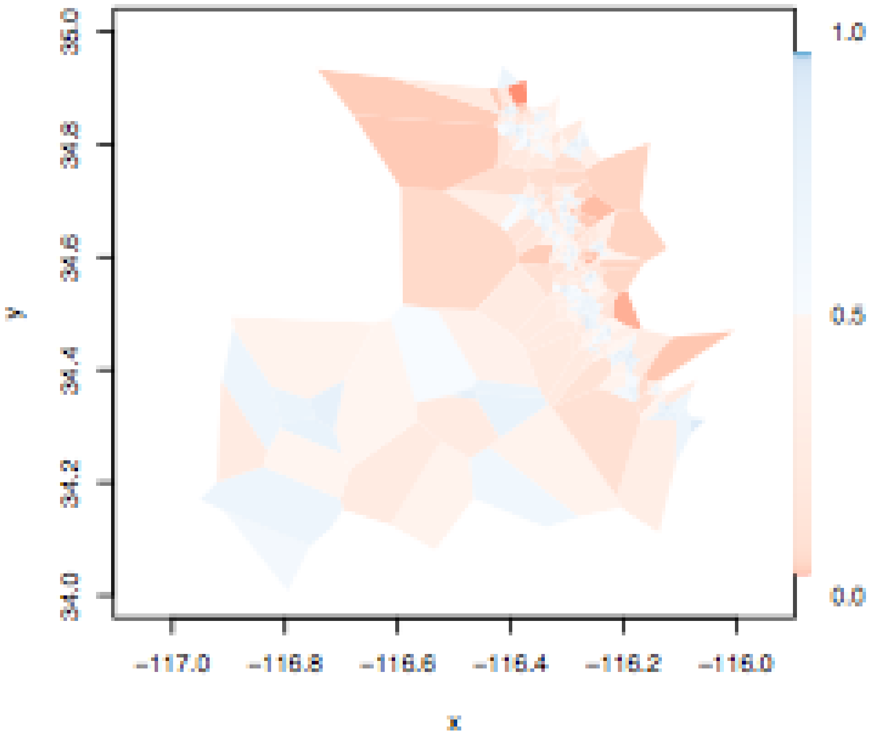
<u>Voronoi residuals</u> (Bray et al. 2013)

$$\hat{r}_i := 1 - \int_{C_i} \hat{\lambda}\,\mathrm{d}\mu = 1 - |C_i|\bar{\lambda},$$

Instead, in each Voronoi cell i, calculate $\text{Energy}_i - E(\text{Energy}_i)$

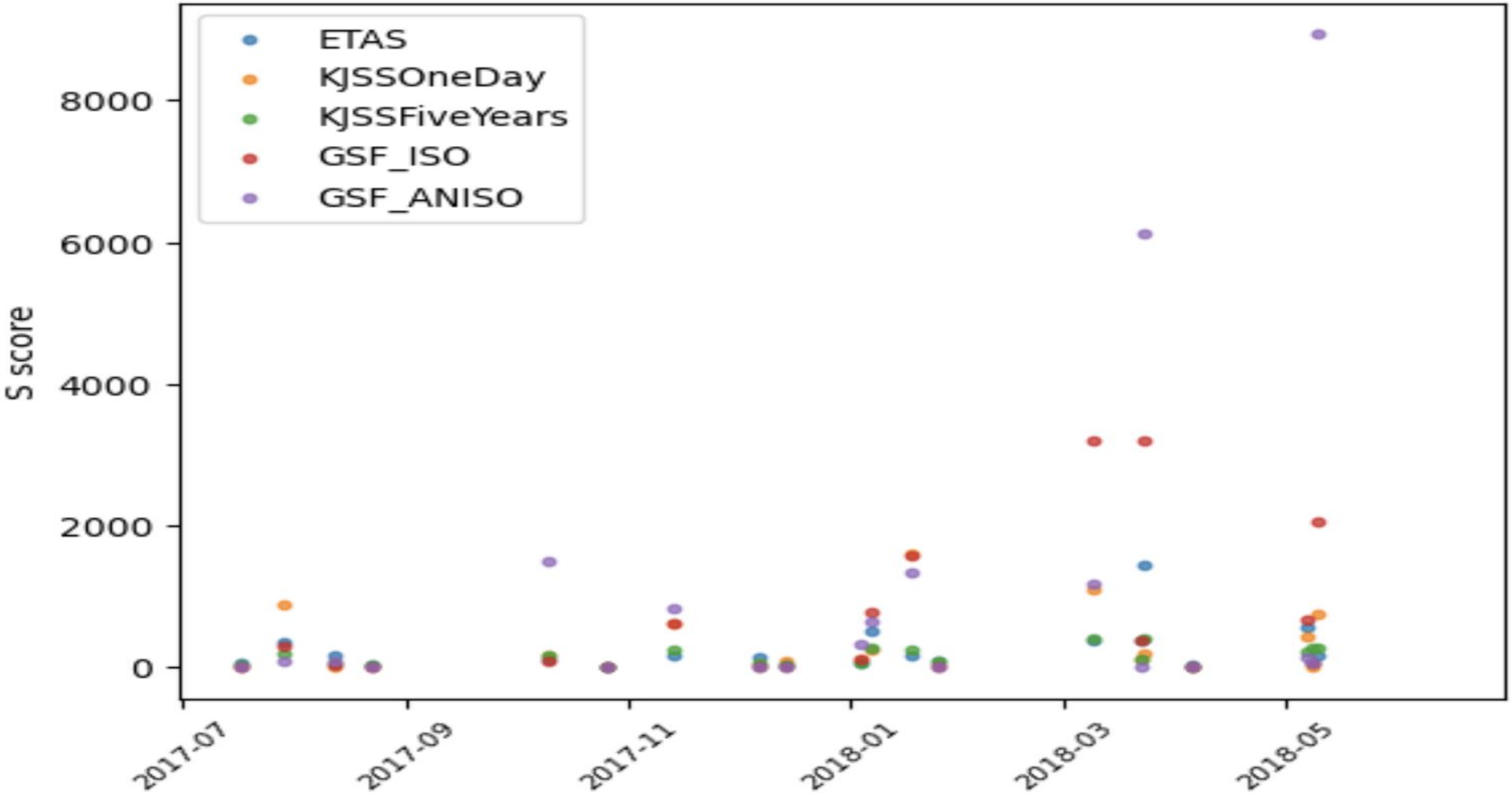
$= 10^{4.8 + 1.5\,M_i} - \int \lambda(t,x,y,m)\ 10^{4.8 + 1.5\,m}\ dt\ dx\ dy\ dm$.





spatially adaptive and nonparametric.

3. Application to CSEP Jun2017-Jun2018.



Conclusions.

1. For fitting and evaluation, more weight should be assigned to the largest events.
2. For loglikelihood measures, instead of $\sum \log(\lambda_i) - \int \lambda(t,x,y,m)\, dt\, dx\, dy\, dm$,

   use $\sum \log(\lambda_i)\, 10^{4.8 + 1.5\, m_i} - \int \lambda(t,x,y,m)\, 10^{4.8 + 1.5m}\, dt\, dx\, dy\, dm$.
3. Residuals should be similarly weighted via $10^{4.8 + 1.5m}$.

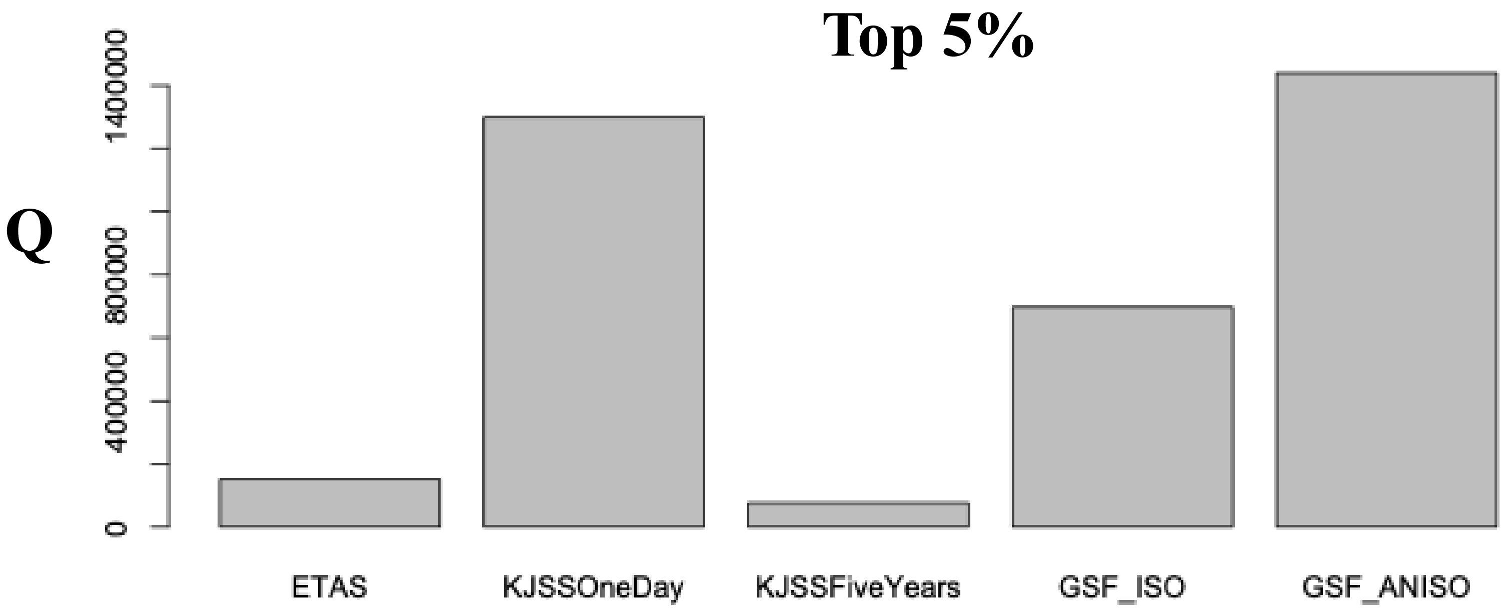
This hopefully will lead to favoring models that help forecast the largest events.

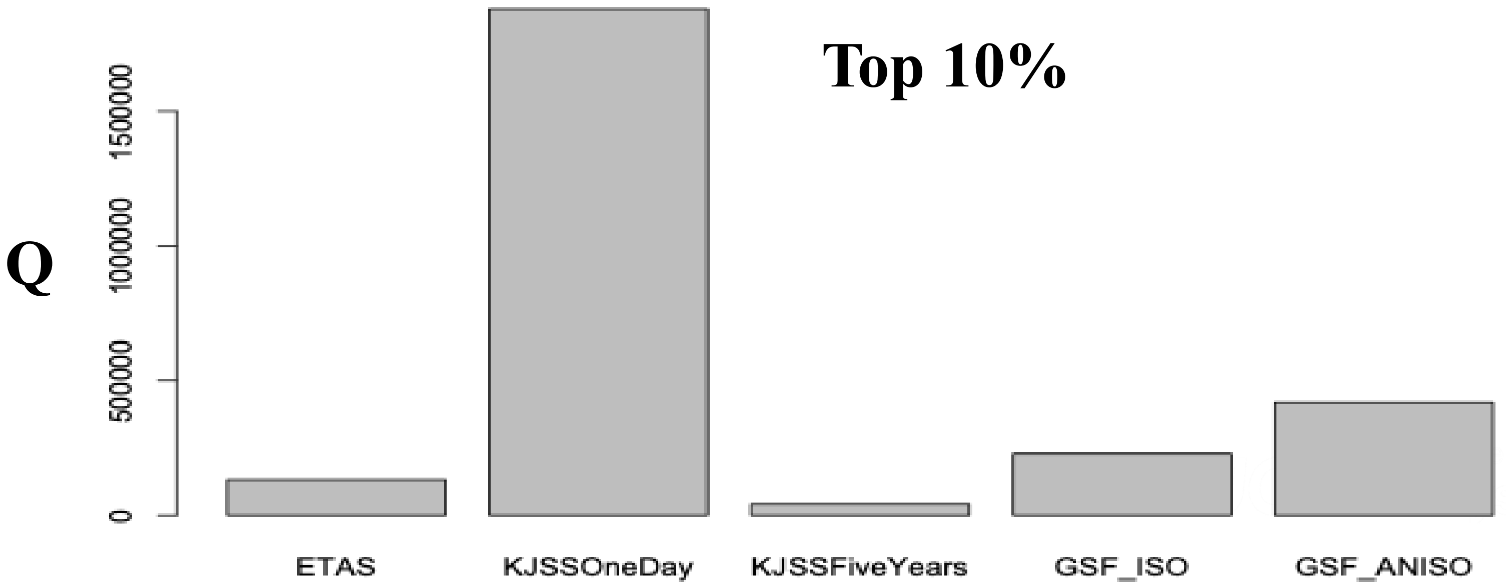
4. Alt. measure is $Q = \mathrm{mean}(\lambda_i(t,x,y): m_i > m^{[.95]}) / \mathrm{mean}(\lambda)$.

Ilya Zaliapin.

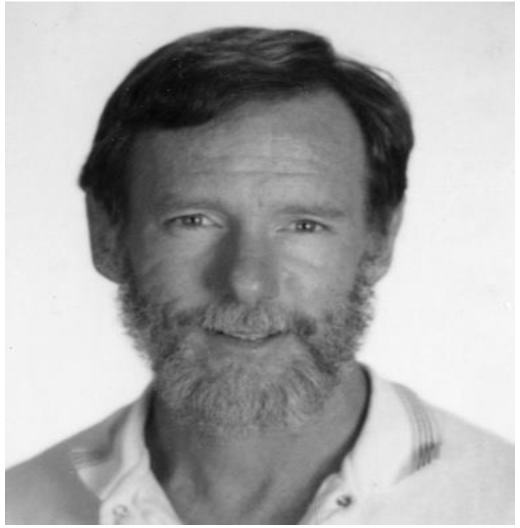
And the winner is ....

Application to CSEP Jun2017-Jun2018.





Yan Kagan



Dave Jackson